

Corrigendum to “Nonparametric Multiplicative Bias
Correction for Kernel-Type Density Estimation on the Unit
Interval” [Comput. Statist. Data Anal. 54 (2010) 473-495]
Appendix: Monte Carlo Results

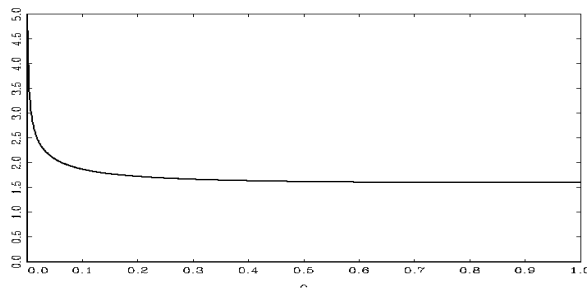
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Upon the correction of the beta-referenced smoothing parameter for the TS-MBC estimator $\hat{b}_{TS,MB}$, Tables 2-5 on pp.481-484 are modified. To implement $\hat{b}_{TS,MB}$, the constant c must be chosen. Because the multiplier $\gamma(c)$ that appears in $MISE^{**} \{ \tilde{f}_{TS,j}(x) \}$ is actually decreasing in $c \in (0, 1)$ as Figure below indicates, there is no optimal c that minimizes the optimal MISE for the estimator. Then, $c = 1/4$ is simply adopted because of the following reasons:

1. This value is very close to $c^* = 0.2636$ that was used for the original simulations.
2. When $c = 1/4$, exponents $(1/(1-c), -c/(1-c))$ in $\tilde{f}_{TS,j}(x) = \{ \hat{f}_{j,b}(x) \}^{1/(1-c)} \{ \hat{f}_{j,b/c}(x) \}^{-c/(1-c)}$ reduce to $(4/3, -1/3)$, which are the values exclusively considered in Terrell and Scott (1980).
3. There is not much difference between $\gamma(1/4) \approx 1.6843$ and $\lim_{c \rightarrow 1} \gamma(c) \approx 1.5922$.

Figure: Plot of $\gamma(c)$



References

- [1] Terrell, G. R., and D. W. Scott (1980): “On Improving Convergence Rates for Nonnegative Kernel Density Estimators,” *Annals of Statistics*, 8, 1160-1163.

Table 2: Average ISE Computed on 1,000 Replications ($n = 100$; Distributions 1-5)

	True Distribution									
	1		2		3		4		5	
	ROT	BR	ROT	BR	ROT	BR	ROT	BR	ROT	BR
Density Estimator:										
\hat{f}_B	0.0283 (0.0000) [0.2006]	0.0072 (0.0000) [0.1217]	0.0382 (0.0030) [0.2316]	0.0408 (0.0202) [0.1766]	0.0321 (0.0027) [0.2103]	0.0416 (0.0256) [0.1456]	0.0298 (0.0009) [0.2073]	0.0382 (0.0281) [0.1299]	0.0319 (0.0073) [0.1961]	0.0338 (0.0112) [0.1865]
\hat{f}_{MB}	0.0308 (0.0000) [0.2004]	0.0057 (0.0000) [0.1217]	0.0410 (0.0009) [0.2342]	0.0324 (0.0093) [0.1796]	0.0335 (0.0010) [0.2122]	0.0451 (0.0207) [0.1472]	0.0310 (0.0001) [0.2069]	0.0494 (0.0337) [0.1297]	0.0255 (0.0033) [0.1892]	0.0249 (0.0054) [0.1787]
\hat{f}_B^r	0.0442 (0.0085) [0.2073]	0.0187 (0.0072) [0.1248]	0.0659 (0.0195) [0.2417]	0.0484 (0.0224) [0.1871]	0.0516 (0.0135) [0.2195]	0.0369 (0.0129) [0.1533]	0.0472 (0.0118) [0.2137]	0.0396 (0.0260) [0.1333]	0.0328 (0.0094) [0.1935]	0.0362 (0.0144) [0.1834]
\hat{f}_{MB}^r	0.0322 (0.0031) [0.1986]	0.0068 (0.0002) [0.1222]	0.0469 (0.0095) [0.2305]	0.0393 (0.0197) [0.1776]	0.0384 (0.0077) [0.2093]	0.0368 (0.0191) [0.1473]	0.0332 (0.0038) [0.2051]	0.0436 (0.0289) [0.1302]	0.0239 (0.0009) [0.1915]	0.0213 (0.0015) [0.1813]
\hat{f}_B^R	0.0282 (0.0000) [0.2007]	0.0068 (0.0000) [0.1217]	0.0384 (0.0017) [0.2333]	0.0359 (0.0137) [0.1792]	0.0323 (0.0021) [0.2116]	0.0394 (0.0219) [0.1475]	0.0297 (0.0009) [0.2073]	0.0383 (0.0284) [0.1297]	0.0324 (0.0091) [0.1933]	0.0352 (0.0137) [0.1832]
\hat{f}_{MB}^R	0.0308 (0.0000) [0.2006]	0.0056 (0.0000) [0.1217]	0.0395 (0.0010) [0.2331]	0.0317 (0.0105) [0.1786]	0.0326 (0.0011) [0.2115]	0.0446 (0.0212) [0.1470]	0.0309 (0.0001) [0.2071]	0.0494 (0.0337) [0.1297]	0.0246 (0.0012) [0.1918]	0.0229 (0.0022) [0.1816]
$\tilde{f}_{TS,B}$	0.0216 (0.0000) [0.1844]	0.0077 (0.0000) [0.1321]	0.0284 (0.0017) [0.2143]	0.0333 (0.0082) [0.1928]	0.0249 (0.0024) [0.1934]	0.0301 (0.0127) [0.1598]	0.0228 (0.0007) [0.1914]	0.0342 (0.0227) [0.1370]	0.0318 (0.0130) [0.1844]	0.0441 (0.0084) [0.1996]
$\tilde{f}_{TS,MB}$	0.0247 (0.0000) [0.1843]	0.0055 (0.0001) [0.1323]	0.0424 (0.0099) [0.2201]	0.0433 (0.0121) [0.1989]	0.0331 (0.0068) [0.1972]	0.0380 (0.0073) [0.1630]	0.0262 (0.0017) [0.1917]	0.0523 (0.0306) [0.1376]	0.0207 (0.0045) [0.1742]	0.0343 (0.0032) [0.1922]
$\tilde{f}_{TS,B}^R$	0.0213 (0.0000) [0.1844]	0.0073 (0.0000) [0.1320]	0.0285 (0.0015) [0.2147]	0.0309 (0.0066) [0.1937]	0.0247 (0.0019) [0.1944]	0.0293 (0.0107) [0.1612]	0.0224 (0.0009) [0.1907]	0.0349 (0.0235) [0.1362]	0.0330 (0.0163) [0.1793]	0.0455 (0.0103) [0.1954]
$\tilde{f}_{TS,MB}^R$	0.0244 (0.0001) [0.1843]	0.0052 (0.0000) [0.1321]	0.0299 (0.0031) [0.2145]	0.0315 (0.0077) [0.1931]	0.0271 (0.0044) [0.1942]	0.0345 (0.0081) [0.1608]	0.0240 (0.0012) [0.1907]	0.0525 (0.0323) [0.1364]	0.0191 (0.0022) [0.1773]	0.0318 (0.0021) [0.1940]
$\tilde{f}_{JLN,B}$	0.0269 (0.0001) [0.1630]	0.0144 (0.0000) [0.1269]	0.0330 (0.0005) [0.1897]	0.0234 (0.0043) [0.1560]	0.0284 (0.0007) [0.1718]	0.0215 (0.0036) [0.1432]	0.0270 (0.0001) [0.1686]	0.0167 (0.0041) [0.1296]	0.0233 (0.0028) [0.1561]	0.0270 (0.0050) [0.1502]
$\tilde{f}_{JLN,MB}$	0.0257 (0.0001) [0.1630]	0.0116 (0.0000) [0.1270]	0.0320 (0.0017) [0.1892]	0.0238 (0.0084) [0.1552]	0.0274 (0.0018) [0.1717]	0.0227 (0.0059) [0.1429]	0.0258 (0.0002) [0.1682]	0.0168 (0.0057) [0.1291]	0.0228 (0.0037) [0.1527]	0.0268 (0.0066) [0.1465]
$\tilde{f}_{JLN,B}^R$	0.0272 (0.0000) [0.1635]	0.0144 (0.0000) [0.1271]	0.0341 (0.0001) [0.1915]	0.0215 (0.0004) [0.1592]	0.0289 (0.0002) [0.1731]	0.0212 (0.0018) [0.1450]	0.0274 (0.0001) [0.1692]	0.0167 (0.0035) [0.1303]	0.0235 (0.0023) [0.1567]	0.0276 (0.0045) [0.1505]
$\tilde{f}_{JLN,MB}^R$	0.0260 (0.0001) [0.1634]	0.0117 (0.0000) [0.1272]	0.0326 (0.0017) [0.1905]	0.0223 (0.0063) [0.1576]	0.0276 (0.0017) [0.1723]	0.0220 (0.0048) [0.1441]	0.0262 (0.0002) [0.1689]	0.0159 (0.0043) [0.1304]	0.0235 (0.0021) [0.1559]	0.0258 (0.0036) [0.1500]
Smoothing Parameter:										
<i>Plain:</i>										
Mean	0.0456	0.4434	0.0304	0.0891	0.0414	0.2049	0.0395	0.3090	0.0299	0.0383
Std. Dev.	0.0021	0.4960	0.0022	0.0282	0.0026	0.1758	0.0024	0.2020	0.0017	0.0064
#(Trimmed)	0	60	0	0	0	3	0	12	0	0
<i>TS-MBC:</i>										
Mean	0.1034	0.4334	0.0690	0.1155	0.0939	0.2242	0.0896	0.3964	0.0677	0.0600
Std. Dev.	0.0047	0.1856	0.0050	0.0627	0.0059	0.1306	0.0055	0.1808	0.0039	0.0377
#(Trimmed)	0	9	0	0	0	2	0	6	0	0
<i>JLN-MBC:</i>										
Mean	0.1034	0.3030	0.0690	0.1461	0.0939	0.2019	0.0896	0.2724	0.0677	0.0848
Std. Dev.	0.0047	0.1154	0.0050	0.0274	0.0059	0.0723	0.0055	0.0847	0.0039	0.0256
#(Trimmed)	0	3	0	0	0	1	0	1	0	0

Note: Corrected parts are highlighted. "ROT" and "BR" in column headings denote "rule-of-thumb" and "beta-referenced" smoothing parameter choice methods. Numbers in parentheses and brackets for density estimators are averages of integrated squared biases and standard errors (defined as square roots of the estimates of asymptotic integrated variances). "Mean", "Std. Dev.", and "#(Trimmed)" for smoothing parameters are averages, standard deviations, and numbers of smoothing parameters trimmed at one.

Table 3: Average ISE Computed on 1,000 Replications ($n = 100$; Distributions 6-10)

	True Distribution									
	6		7		8		9		10	
	ROT	BR	ROT	BR	ROT	BR	ROT	BR	ROT	BR
Density Estimator:										
\hat{f}_B	0.2801 (0.2524) [0.2015]	0.3090 (0.2837) [0.1843]	0.0310 (0.0023) [0.2029]	0.0255 (0.0104) [0.1502]	0.0384 (0.0102) [0.2106]	0.0449 (0.0210) [0.1906]	0.0387 (0.0122) [0.1977]	0.0542 (0.0446) [0.1398]	0.1004 (0.0728) [0.2089]	0.1336 (0.1147) [0.1717]
\hat{f}_{MB}	0.2850 (0.2496) [0.2137]	0.3184 (0.2837) [0.1948]	0.0337 (0.0014) [0.2060]	0.0271 (0.0100) [0.1528]	0.0331 (0.0063) [0.2044]	0.0379 (0.0155) [0.1844]	0.0356 (0.0101) [0.1920]	0.0604 (0.0510) [0.1379]	0.1149 (0.0861) [0.2068]	0.1657 (0.1448) [0.1738]
\hat{f}_B^r	0.1382 (0.0700) [0.2370]	0.1533 (0.0893) [0.2174]	0.0506 (0.0112) [0.2136]	0.0291 (0.0051) [0.1599]	0.0430 (0.0148) [0.2092]	0.0568 (0.0315) [0.1891]	0.0453 (0.0183) [0.1968]	0.0686 (0.0586) [0.1391]	0.1140 (0.0837) [0.2110]	0.1545 (0.1341) [0.1758]
\hat{f}_{MB}^r	0.2422 (0.2009) [0.2177]	0.2708 (0.2325) [0.1996]	0.0377 (0.0071) [0.2028]	0.0236 (0.0097) [0.1527]	0.0315 (0.0049) [0.2051]	0.0356 (0.0138) [0.1848]	0.0360 (0.0114) [0.1915]	0.0637 (0.0522) [0.1369]	0.1222 (0.0957) [0.2029]	0.1637 (0.1467) [0.1691]
\hat{f}_B^R	0.2550 (0.2194) [0.2139]	0.2815 (0.2482) [0.1966]	0.0316 (0.0015) [0.2051]	0.0243 (0.0079) [0.1533]	0.0385 (0.0115) [0.2083]	0.0461 (0.0234) [0.1877]	0.0375 (0.0124) [0.1951]	0.0527 (0.0442) [0.1371]	0.0969 (0.0699) [0.2080]	0.1358 (0.1165) [0.1723]
\hat{f}_{MB}^R	0.2842 (0.2483) [0.2149]	0.3169 (0.2817) [0.1966]	0.0326 (0.0014) [0.2047]	0.0267 (0.0102) [0.1524]	0.0327 (0.0048) [0.2062]	0.0366 (0.0133) [0.1861]	0.0359 (0.0097) [0.1931]	0.0602 (0.0510) [0.1372]	0.1086 (0.0809) [0.2051]	0.1505 (0.1315) [0.1702]
$\tilde{f}_{TS,B}$	0.3240 (0.3035) [0.1806]	0.3110 (0.2854) [0.1870]	0.0256 (0.0036) [0.1854]	0.0256 (0.0087) [0.1600]	0.0414 (0.0195) [0.1981]	0.0496 (0.0088) [0.2221]	0.0449 (0.0242) [0.1851]	0.0631 (0.0575) [0.1325]	0.1343 (0.1128) [0.1960]	0.1452 (0.1240) [0.1865]
$\tilde{f}_{TS,MB}$	0.3423 (0.3153) [0.1917]	0.3363 (0.2972) [0.1993]	0.0299 (0.0038) [0.1899]	0.0294 (0.0091) [0.1628]	0.0362 (0.0149) [0.1911]	0.0454 (0.0058) [0.2163]	0.0492 (0.0279) [0.1805]	0.0668 (0.0637) [0.1323]	0.1964 (0.1720) [0.2003]	0.2147 (0.1877) [0.1916]
$\tilde{f}_{TS,B}^R$	0.2934 (0.2664) [0.1934]	0.2839 (0.2503) [0.1996]	0.0255 (0.0023) [0.1881]	0.0244 (0.0061) [0.1634]	0.0409 (0.0212) [0.1934]	0.0495 (0.0096) [0.2187]	0.0432 (0.0242) [0.1818]	0.0608 (0.0560) [0.1296]	0.1254 (0.1053) [0.1941]	0.1398 (0.1196) [0.1854]
$\tilde{f}_{TS,MB}^R$	0.3410 (0.3144) [0.1924]	0.3337 (0.2977) [0.1989]	0.0273 (0.0033) [0.1873]	0.0284 (0.0094) [0.1622]	0.0360 (0.0146) [0.1916]	0.0455 (0.0056) [0.2167]	0.0492 (0.0279) [0.1807]	0.0656 (0.0628) [0.1302]	0.1438 (0.1250) [0.1915]	0.1605 (0.1402) [0.1829]
$\tilde{f}_{JLN,B}$	0.2787 (0.2519) [0.1648]	0.2617 (0.2280) [0.1739]	0.0291 (0.0014) [0.1659]	0.0262 (0.0029) [0.1495]	0.0316 (0.0070) [0.1693]	0.0373 (0.0136) [0.1588]	0.0383 (0.0137) [0.1603]	0.0512 (0.0383) [0.1312]	0.1011 (0.0761) [0.1706]	0.1163 (0.0951) [0.1601]
$\tilde{f}_{JLN,MB}$	0.3182 (0.2948) [0.1663]	0.3007 (0.2693) [0.1758]	0.0270 (0.0022) [0.1661]	0.0255 (0.0043) [0.1496]	0.0346 (0.0105) [0.1664]	0.0416 (0.0185) [0.1558]	0.0433 (0.0202) [0.1585]	0.0615 (0.0511) [0.1308]	0.1123 (0.0902) [0.1692]	0.1324 (0.1121) [0.1593]
$\tilde{f}_{JLN,B}^R$	0.2555 (0.2210) [0.1748]	0.2423 (0.1996) [0.1838]	0.0298 (0.0006) [0.1678]	0.0266 (0.0017) [0.1517]	0.0319 (0.0067) [0.1696]	0.0376 (0.0133) [0.1590]	0.0380 (0.0139) [0.1593]	0.0505 (0.0383) [0.1296]	0.1089 (0.0825) [0.1722]	0.1265 (0.1035) [0.1622]
$\tilde{f}_{JLN,MB}^R$	0.3103 (0.2830) [0.1724]	0.2952 (0.2590) [0.1817]	0.0272 (0.0021) [0.1667]	0.0256 (0.0041) [0.1504]	0.0351 (0.0091) [0.1689]	0.0409 (0.0158) [0.1586]	0.0436 (0.0200) [0.1589]	0.0613 (0.0511) [0.1301]	0.1170 (0.0941) [0.1703]	0.1357 (0.1147) [0.1600]
Smoothing Parameter:										
<i>Plain:</i>										
Mean	0.0559	0.0771	0.0502	0.1802	0.0282	0.0459	0.0424	0.2372	0.0453	0.1035
Std. Dev.	0.0021	0.0159	0.0020	0.1420	0.0019	0.0124	0.0019	0.1892	0.0027	0.0256
#(Trimmed)	0	0	0	7	0	0	0	10	0	0
<i>TS-MBC:</i>										
Mean	0.1268	0.1171	0.1137	0.2113	0.0639	0.0484	0.0961	0.4238	0.1026	0.1265
Std. Dev.	0.0048	0.0222	0.0045	0.0951	0.0042	0.0361	0.0043	0.1305	0.0061	0.0233
#(Trimmed)	0	0	0	1	0	0	0	5	0	0
<i>JLN-MBC:</i>										
Mean	0.1268	0.1084	0.1137	0.1774	0.0639	0.0879	0.0961	0.2440	0.1026	0.1355
Std. Dev.	0.0048	0.0133	0.0045	0.0606	0.0042	0.0200	0.0043	0.0623	0.0061	0.0199
#(Trimmed)	0	0	0	0	0	0	0	0	0	0

Note: Corrected parts are highlighted. "ROT" and "BR" in column headings denote "rule-of-thumb" and "beta-referenced" smoothing parameter choice methods. Numbers in parentheses and brackets for density estimators are averages of integrated squared biases and standard errors (defined as square roots of the estimates of asymptotic integrated variances). "Mean", "Std. Dev.", and "#(Trimmed)" for smoothing parameters are averages, standard deviations, and numbers of smoothing parameters trimmed at one.

Table 4: Average ISE Computed on 1,000 Replications ($n = 200$; Distributions 1-5)

	True Distribution									
	1		2		3		4		5	
	ROT	BR	ROT	BR	ROT	BR	ROT	BR	ROT	BR
Density Estimator:										
\hat{f}_B	0.0172 (0.0000) [0.1520]	0.0036 (0.0000) [0.0860]	0.0231 (0.0020) [0.1757]	0.0257 (0.0146) [0.1323]	0.0198 (0.0019) [0.1596]	0.0270 (0.0176) [0.1109]	0.0182 (0.0006) [0.1569]	0.0296 (0.0245) [0.0942]	0.0192 (0.0044) [0.1474]	0.0202 (0.0069) [0.1397]
\hat{f}_{MB}	0.0188 (0.0000) [0.1519]	0.0029 (0.0000) [0.0860]	0.0241 (0.0003) [0.1775]	0.0173 (0.0053) [0.1345]	0.0206 (0.0006) [0.1608]	0.0246 (0.0113) [0.1122]	0.0191 (0.0001) [0.1566]	0.0347 (0.0260) [0.0941]	0.0156 (0.0020) [0.1433]	0.0150 (0.0032) [0.1349]
\hat{f}_B^r	0.0283 (0.0070) [0.1567]	0.0135 (0.0072) [0.0882]	0.0461 (0.0191) [0.1828]	0.0389 (0.0252) [0.1399]	0.0349 (0.0119) [0.1660]	0.0261 (0.0132) [0.1168]	0.0305 (0.0092) [0.1613]	0.0323 (0.0254) [0.0969]	0.0199 (0.0056) [0.1459]	0.0217 (0.0089) [0.1378]
\hat{f}_{MB}^r	0.0204 (0.0026) [0.1504]	0.0038 (0.0002) [0.0864]	0.0293 (0.0070) [0.1748]	0.0276 (0.0168) [0.1327]	0.0255 (0.0066) [0.1586]	0.0227 (0.0137) [0.1118]	0.0214 (0.0033) [0.1552]	0.0299 (0.0222) [0.0945]	0.0147 (0.0005) [0.1447]	0.0129 (0.0008) [0.1365]
\hat{f}_B^R	0.0171 (0.0000) [0.1521]	0.0034 (0.0000) [0.0860]	0.0228 (0.0011) [0.1768]	0.0215 (0.0097) [0.1340]	0.0198 (0.0015) [0.1604]	0.0249 (0.0148) [0.1122]	0.0181 (0.0006) [0.1569]	0.0299 (0.0248) [0.0940]	0.0197 (0.0055) [0.1458]	0.0214 (0.0086) [0.1378]
\hat{f}_{MB}^R	0.0188 (0.0000) [0.1520]	0.0029 (0.0000) [0.0860]	0.0234 (0.0004) [0.1768]	0.0173 (0.0061) [0.1337]	0.0202 (0.0007) [0.1604]	0.0244 (0.0117) [0.1119]	0.0190 (0.0001) [0.1568]	0.0347 (0.0260) [0.0941]	0.0149 (0.0007) [0.1448]	0.0135 (0.0011) [0.1367]
$\tilde{f}_{TS,B}$	0.0121 (0.0000) [0.1355]	0.0040 (0.0000) [0.0938]	0.0160 (0.0013) [0.1573]	0.0182 (0.0056) [0.1410]	0.0144 (0.0018) [0.1422]	0.0183 (0.0089) [0.1188]	0.0129 (0.0005) [0.1404]	0.0280 (0.0213) [0.0978]	0.0196 (0.0091) [0.1343]	0.0265 (0.0068) [0.1423]
$\tilde{f}_{TS,MB}$	0.0140 (0.0000) [0.1354]	0.0028 (0.0000) [0.0939]	0.0244 (0.0066) [0.1610]	0.0251 (0.0117) [0.1456]	0.0199 (0.0052) [0.1447]	0.0217 (0.0064) [0.1213]	0.0148 (0.0011) [0.1404]	0.0433 (0.0275) [0.0982]	0.0117 (0.0025) [0.1277]	0.0188 (0.0021) [0.1372]
$\tilde{f}_{TS,B}^R$	0.0120 (0.0000) [0.1355]	0.0037 (0.0000) [0.0938]	0.0159 (0.0011) [0.1576]	0.0167 (0.0045) [0.1416]	0.0142 (0.0014) [0.1429]	0.0173 (0.0074) [0.1197]	0.0127 (0.0006) [0.1400]	0.0289 (0.0222) [0.0973]	0.0211 (0.0116) [0.1311]	0.0282 (0.0084) [0.1395]
$\tilde{f}_{TS,MB}^R$	0.0139 (0.0000) [0.1355]	0.0027 (0.0000) [0.0938]	0.0171 (0.0019) [0.1577]	0.0156 (0.0051) [0.1414]	0.0163 (0.0032) [0.1428]	0.0189 (0.0058) [0.1195]	0.0139 (0.0008) [0.1400]	0.0440 (0.0290) [0.0974]	0.0107 (0.0012) [0.1296]	0.0169 (0.0012) [0.1384]
$\tilde{f}_{JLN,B}$	0.0151 (0.0000) [0.1199]	0.0073 (0.0000) [0.0899]	0.0182 (0.0003) [0.1395]	0.0131 (0.0032) [0.1137]	0.0161 (0.0005) [0.1264]	0.0125 (0.0024) [0.1059]	0.0152 (0.0001) [0.1239]	0.0096 (0.0034) [0.0933]	0.0133 (0.0017) [0.1144]	0.0148 (0.0031) [0.1097]
$\tilde{f}_{JLN,MB}$	0.0147 (0.0001) [0.1199]	0.0060 (0.0000) [0.0899]	0.0178 (0.0008) [0.1392]	0.0139 (0.0060) [0.1132]	0.0160 (0.0013) [0.1264]	0.0141 (0.0045) [0.1058]	0.0148 (0.0001) [0.1236]	0.0094 (0.0040) [0.0930]	0.0130 (0.0020) [0.1124]	0.0140 (0.0038) [0.1074]
$\tilde{f}_{JLN,B}^R$	0.0152 (0.0000) [0.1202]	0.0072 (0.0000) [0.0900]	0.0185 (0.0000) [0.1405]	0.0110 (0.0003) [0.1156]	0.0162 (0.0002) [0.1271]	0.0120 (0.0013) [0.1070]	0.0153 (0.0000) [0.1241]	0.0094 (0.0029) [0.0937]	0.0132 (0.0013) [0.1148]	0.0149 (0.0027) [0.1100]
$\tilde{f}_{JLN,MB}^R$	0.0148 (0.0000) [0.1201]	0.0060 (0.0000) [0.0900]	0.0180 (0.0009) [0.1399]	0.0128 (0.0048) [0.1146]	0.0160 (0.0012) [0.1267]	0.0138 (0.0041) [0.1064]	0.0149 (0.0001) [0.1240]	0.0085 (0.0030) [0.0938]	0.0132 (0.0012) [0.1142]	0.0130 (0.0022) [0.1095]
Smoothing Parameter:										
<i>Plain:</i>										
Mean	0.0346	0.4448	0.0232	0.0699	0.0315	0.1439	0.0300	0.2637	0.0227	0.0291
Std. Dev.	0.0011	0.3946	0.0012	0.0146	0.0014	0.0950	0.0013	0.1340	0.0009	0.0034
#{Trimmed}	0	62	0	0	0	2	0	7	0	0
<i>TS-MBC:</i>										
Mean	0.0887	0.4250	0.0595	0.0959	0.0807	0.1720	0.0770	0.3714	0.0582	0.0537
Std. Dev.	0.0029	0.1813	0.0031	0.0361	0.0036	0.0698	0.0034	0.1490	0.0024	0.0319
#{Trimmed}	0	16	0	0	0	0	0	0	0	0
<i>JLN-MBC:</i>										
Mean	0.0887	0.3023	0.0595	0.1303	0.0807	0.1657	0.0770	0.2503	0.0582	0.0716
Std. Dev.	0.0029	0.1137	0.0031	0.0179	0.0036	0.0467	0.0034	0.0635	0.0024	0.0145
#{Trimmed}	0	2	0	0	0	0	0	0	0	0

Note: Corrected parts are highlighted. "ROT" and "BR" in column headings denote "rule-of-thumb" and "beta-referenced" smoothing parameter choice methods. Numbers in parentheses and brackets for density estimators are averages of integrated squared biases and standard errors (defined as square roots of the estimates of asymptotic integrated variances). "Mean", "Std. Dev.", and "#{Trimmed}" for smoothing parameters are averages, standard deviations, and numbers of smoothing parameters trimmed at one.

Table 5: Average ISE Computed on 1,000 Replications ($n = 200$; Distributions 6 - 10)

	True Distribution									
	6		7		8		9		10	
	ROT	BR	ROT	BR	ROT	BR	ROT	BR	ROT	BR
Density Estimator:										
\hat{f}_B	0.2356 (0.2186) [0.1552]	0.2662 (0.2507) [0.1421]	0.0184 (0.0015) [0.1544]	0.0168 (0.0080) [0.1148]	0.0237 (0.0064) [0.1585]	0.0280 (0.0141) [0.1431]	0.0247 (0.0084) [0.1486]	0.0457 (0.0393) [0.1064]	0.0794 (0.0629) [0.1574]	0.1077 (0.0969) [0.1307]
\hat{f}_{MB}	0.2355 (0.2133) [0.1650]	0.2691 (0.2476) [0.1508]	0.0198 (0.0007) [0.1565]	0.0172 (0.0068) [0.1169]	0.0198 (0.0033) [0.1544]	0.0219 (0.0090) [0.1386]	0.0219 (0.0061) [0.1444]	0.0514 (0.0446) [0.1047]	0.0884 (0.0714) [0.1548]	0.1329 (0.1214) [0.1313]
\hat{f}_B^r	0.0946 (0.0539) [0.1821]	0.1068 (0.0694) [0.1674]	0.0337 (0.0112) [0.1619]	0.0206 (0.0079) [0.1222]	0.0262 (0.0091) [0.1576]	0.0353 (0.0208) [0.1420]	0.0287 (0.0123) [0.1480]	0.0619 (0.0548) [0.1060]	0.0862 (0.0686) [0.1584]	0.1263 (0.1152) [0.1331]
\hat{f}_{MB}^r	0.1918 (0.1668) [0.1675]	0.2219 (0.1986) [0.1537]	0.0237 (0.0057) [0.1540]	0.0178 (0.0097) [0.1162]	0.0187 (0.0023) [0.1550]	0.0196 (0.0073) [0.1390]	0.0221 (0.0067) [0.1443]	0.0551 (0.0465) [0.1039]	0.0968 (0.0810) [0.1526]	0.1381 (0.1278) [0.1278]
\hat{f}_B^R	0.2106 (0.1891) [0.1640]	0.2383 (0.2183) [0.1509]	0.0185 (0.0010) [0.1558]	0.0155 (0.0060) [0.1169]	0.0239 (0.0073) [0.1571]	0.0291 (0.0157) [0.1413]	0.0242 (0.0086) [0.1470]	0.0448 (0.0390) [0.1044]	0.0752 (0.0591) [0.1566]	0.1073 (0.0966) [0.1306]
\hat{f}_{MB}^R	0.2350 (0.2127) [0.1655]	0.2681 (0.2466) [0.1516]	0.0192 (0.0007) [0.1557]	0.0169 (0.0070) [0.1164]	0.0194 (0.0024) [0.1556]	0.0207 (0.0073) [0.1399]	0.0219 (0.0058) [0.1453]	0.0514 (0.0447) [0.1043]	0.0857 (0.0690) [0.1543]	0.1209 (0.1101) [0.1290]
$\hat{f}_{TS,B}$	0.2935 (0.2820) [0.1341]	0.2789 (0.2644) [0.1394]	0.0145 (0.0027) [0.1368]	0.0165 (0.0073) [0.1184]	0.0272 (0.0149) [0.1446]	0.0283 (0.0073) [0.1590]	0.0322 (0.0202) [0.1354]	0.0579 (0.0544) [0.0987]	0.1156 (0.1037) [0.1440]	0.1251 (0.1136) [0.1378]
$\hat{f}_{TS,MB}$	0.3099 (0.2943) [0.1431]	0.2996 (0.2770) [0.1492]	0.0171 (0.0029) [0.1402]	0.0188 (0.0071) [0.1208]	0.0251 (0.0111) [0.1395]	0.0252 (0.0046) [0.1547]	0.0350 (0.0229) [0.1316]	0.0634 (0.0610) [0.0984]	0.1655 (0.1520) [0.1462]	0.1856 (0.1715) [0.1409]
$\hat{f}_{TS,B}^R$	0.2623 (0.2472) [0.1431]	0.2505 (0.2316) [0.1483]	0.0142 (0.0017) [0.1385]	0.0150 (0.0051) [0.1207]	0.0273 (0.0161) [0.1416]	0.0286 (0.0079) [0.1567]	0.0313 (0.0202) [0.1330]	0.0560 (0.0530) [0.0965]	0.1048 (0.0938) [0.1422]	0.1172 (0.1064) [0.1364]
$\hat{f}_{TS,MB}^R$	0.3093 (0.2944) [0.1428]	0.2981 (0.2776) [0.1483]	0.0151 (0.0021) [0.1381]	0.0181 (0.0072) [0.1200]	0.0228 (0.0106) [0.1401]	0.0252 (0.0044) [0.1552]	0.0351 (0.0227) [0.1321]	0.0624 (0.0602) [0.0968]	0.1217 (0.1110) [0.1404]	0.1363 (0.1253) [0.1347]
$\hat{f}_{JLN,B}$	0.2455 (0.2305) [0.1225]	0.2268 (0.2079) [0.1294]	0.0159 (0.0009) [0.1224]	0.0147 (0.0022) [0.1106]	0.0186 (0.0048) [0.1241]	0.0227 (0.0099) [0.1160]	0.0246 (0.0104) [0.1173]	0.0413 (0.0334) [0.0969]	0.0811 (0.0671) [0.1251]	0.0944 (0.0828) [0.1179]
$\hat{f}_{JLN,MB}$	0.2857 (0.2724) [0.1237]	0.2658 (0.2481) [0.1309]	0.0151 (0.0014) [0.1225]	0.0148 (0.0032) [0.1107]	0.0208 (0.0072) [0.1222]	0.0260 (0.0136) [0.1140]	0.0290 (0.0155) [0.1158]	0.0532 (0.0465) [0.0965]	0.0927 (0.0800) [0.1238]	0.1089 (0.0978) [0.1171]
$\hat{f}_{JLN,B}^R$	0.2221 (0.2031) [0.1293]	0.2064 (0.1829) [0.1362]	0.0160 (0.0005) [0.1235]	0.0145 (0.0013) [0.1119]	0.0187 (0.0047) [0.1242]	0.0228 (0.0098) [0.1161]	0.0245 (0.0107) [0.1165]	0.0410 (0.0336) [0.0957]	0.0855 (0.0710) [0.1258]	0.1009 (0.0886) [0.1190]
$\hat{f}_{JLN,MB}^R$	0.2779 (0.2626) [0.1279]	0.2597 (0.2396) [0.1349]	0.0151 (0.0013) [0.1228]	0.0148 (0.0031) [0.1110]	0.0209 (0.0064) [0.1237]	0.0251 (0.0119) [0.1157]	0.0290 (0.0153) [0.1162]	0.0531 (0.0465) [0.0960]	0.0971 (0.0839) [0.1246]	0.1117 (0.1002) [0.1176]
Smoothing Parameter:										
<i>Plain:</i>										
Mean	0.0424	0.0572	0.0380	0.1223	0.0213	0.0343	0.0321	0.1591	0.0344	0.0761
Std. Dev.	0.0011	0.0072	0.0010	0.0499	0.0010	0.0058	0.0010	0.0983	0.0014	0.0116
#(Trimmed)	0	0	0	0	0	0	0	3	0	0
<i>TS-MBC:</i>										
Mean	0.1088	0.0977	0.0975	0.1702	0.0547	0.0407	0.0823	0.3367	0.0882	0.1059
Std. Dev.	0.0028	0.0112	0.0027	0.0447	0.0025	0.0223	0.0026	0.0881	0.0037	0.0122
#(Trimmed)	0	0	0	0	0	0	0	3	0	0
<i>JLN-MBC:</i>										
Mean	0.1088	0.0919	0.0975	0.1461	0.0547	0.0749	0.0823	0.2003	0.0882	0.1142
Std. Dev.	0.0028	0.0071	0.0027	0.0303	0.0025	0.0115	0.0026	0.0395	0.0037	0.0107
#(Trimmed)	0	0	0	0	0	0	0	0	0	0

Note: Corrected parts are highlighted. "ROT" and "BR" in column headings denote "rule-of-thumb" and "beta-referenced" smoothing parameter choice methods. Numbers in parentheses and brackets for density estimators are averages of integrated squared biases and standard errors (defined as square roots of the estimates of asymptotic integrated variances). "Mean", "Std. Dev.", and "#(Trimmed)" for smoothing parameters are averages, standard deviations, and numbers of smoothing parameters trimmed at one.