

Supplement to
“Functional-Coefficient Cointegration Models
in the Presence of Deterministic Trends”:
Monte Carlo Results

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Table S1 below presents simulation results in Section 6. While we provide only sample medians of RMSEs, there is not much difference in tendencies from their (unreported) sample averages. “Restricted” and “Unrestricted” denote the regression with no linear trend (9) and the regression with a linear trend (10), respectively. For each functional form of $\beta(z)$, the cases with $\rho = \sigma_{21} = \phi_{11} = 0.4$ are chosen as benchmarks, the results of which are put in frames. Accordingly, each panel reports the results when a parameter is changed with all others held constant. Furthermore, for each combination of the model (i.e., Restricted or Unrestricted), $\beta(z)$ and the sample size, the smallest performance measure is typed in bold faces.

Table S1: Median RMSEs for LL and PLLR Estimators

Panel A: $\beta(z) = 1$

	$T = 100$						$T = 250$					
	<i>Restricted</i>			<i>Unrestricted</i>			<i>Restricted</i>			<i>Unrestricted</i>		
	LL			LL			LL			LL		
	SP	ROT	PLLR	SP	ROT	PLLR	SP	ROT	PLLR	SP	ROT	PLLR
ρ	($\sigma_{21} = \varphi_{11} = 0.4$)											
-0.8	0.3303	0.5247	0.2233	0.4882	0.2843	0.2655	0.0267	0.2303	0.0588	0.0912	0.0854	0.1467
-0.4	0.0933	0.3550	0.1504	0.2314	0.1833	0.2312	0.0162	0.0573	0.0332	0.0632	0.0693	0.0969
0.0	0.0801	0.3271	0.1386	0.2064	0.1829	0.2266	0.0148	0.0470	0.0289	0.0607	0.0685	0.0805
0.4	0.1017	0.3652	0.1587	0.2339	0.1970	0.2348	0.0160	0.0613	0.0334	0.0661	0.0703	0.0909
0.8	0.3926	0.5727	0.2613	0.6520	0.3312	0.2821	0.0324	0.2392	0.0646	0.1016	0.0895	0.1577
σ_{21}	($\rho = \varphi_{11} = 0.4$)											
-0.8	0.1171	0.3743	0.1698	0.2800	0.2197	0.2318	0.0163	0.0647	0.0336	0.0789	0.0833	0.0917
-0.4	0.1098	0.3740	0.1668	0.2425	0.2026	0.2361	0.0161	0.0672	0.0339	0.0658	0.0728	0.0901
0.0	0.1036	0.3722	0.1615	0.2236	0.1905	0.2439	0.0159	0.0620	0.0337	0.0626	0.0701	0.0908
0.4	0.1017	0.3652	0.1587	0.2339	0.1970	0.2348	0.0160	0.0613	0.0334	0.0661	0.0703	0.0909
0.8	0.1073	0.3827	0.1626	0.2661	0.2226	0.2327	0.0160	0.0615	0.0328	0.0776	0.0804	0.0932
φ_{11}	($\rho = \sigma_{21} = 0.4$)											
-0.8	0.1400	0.4435	0.2388	0.3083	0.2627	0.3588	0.0220	0.0910	0.0498	0.0828	0.0962	0.1336
-0.4	0.0943	0.3810	0.1603	0.2159	0.1769	0.2441	0.0151	0.0561	0.0321	0.0584	0.0672	0.0960
0.0	0.0953	0.3611	0.1424	0.2053	0.1746	0.2373	0.0141	0.0535	0.0303	0.0555	0.0644	0.0864
0.4	0.1017	0.3652	0.1587	0.2339	0.1970	0.2348	0.0160	0.0613	0.0334	0.0661	0.0703	0.0909
0.8	0.1599	0.4293	0.2272	0.3863	0.3119	0.3330	0.0265	0.0843	0.0512	0.1243	0.1295	0.1426

Panel B: $\beta(z) = 0.3 - 0.5\exp(-1.25z^2)$

	$T = 100$						$T = 250$					
	<i>Restricted</i>			<i>Unrestricted</i>			<i>Restricted</i>			<i>Unrestricted</i>		
	LL			LL			LL			LL		
	SP	ROT	PLLR	SP	ROT	PLLR	SP	ROT	PLLR	SP	ROT	PLLR
ρ	($\sigma_{21} = \varphi_{11} = 0.4$)											
-0.8	0.3653	0.5906	0.4728	2.1139	1.7938	1.4716	0.1901	0.3983	0.3125	1.5740	1.8799	2.7853
-0.4	0.2015	0.4574	0.3447	1.3191	1.4330	1.7825	0.1531	0.2366	0.2098	1.1154	1.4865	1.9576
0.0	0.1926	0.4476	0.3437	1.2849	1.5045	1.8315	0.1485	0.2311	0.2011	1.0507	1.4371	1.6559
0.4	0.2183	0.4753	0.3665	1.4481	1.5343	1.8642	0.1501	0.2395	0.2078	1.0870	1.4601	1.8954
0.8	0.3951	0.6544	0.5117	2.8966	2.3439	1.7781	0.1994	0.3730	0.2900	1.4396	1.5878	2.5937
σ_{21}	($\rho = \varphi_{11} = 0.4$)											
-0.8	0.2004	0.4962	0.3699	1.4046	1.5102	1.9217	0.1495	0.2412	0.2086	1.0739	1.4432	1.8937
-0.4	0.2038	0.5205	0.3714	1.3727	1.5127	1.8618	0.1492	0.2418	0.2097	1.0565	1.4303	1.8621
0.0	0.2026	0.4981	0.3789	1.4285	1.5241	1.8428	0.1503	0.2419	0.2102	1.0656	1.4336	1.8896
0.4	0.2183	0.4753	0.3665	1.4481	1.5343	1.8642	0.1501	0.2395	0.2078	1.0870	1.4601	1.8954
0.8	0.2133	0.5160	0.3913	1.4997	1.5806	1.8876	0.1518	0.2429	0.2121	1.0766	1.4471	1.8710
φ_{11}	($\rho = \sigma_{21} = 0.4$)											
-0.8	0.2299	0.5692	0.4378	1.4184	1.5584	1.9583	0.1523	0.2513	0.2140	1.0784	1.4450	1.9405
-0.4	0.2069	0.5031	0.3818	1.4304	1.5161	1.9683	0.1481	0.2394	0.2086	1.0850	1.4636	1.8931
0.0	0.2034	0.5003	0.3772	1.4502	1.5098	1.9734	0.1491	0.2387	0.2082	1.0888	1.4598	1.8885
0.4	0.2183	0.4753	0.3665	1.4481	1.5343	1.8642	0.1501	0.2395	0.2078	1.0870	1.4601	1.8954
0.8	0.2390	0.5435	0.4223	1.4892	1.5467	1.9616	0.1532	0.2446	0.2118	1.0894	1.4600	1.8982

Table S1: *Continued*

Panel C: $\beta(z) = 0.5/\{1+\exp(-4z)\} - 0.75$

	$T = 100$						$T = 250$					
	<i>Restricted</i>			<i>Unrestricted</i>			<i>Restricted</i>			<i>Unrestricted</i>		
	LL			LL			LL			LL		
	SP	ROT	PLLR	SP	ROT	PLLR	SP	ROT	PLLR	SP	ROT	PLLR
ρ	($\sigma_{21} = \varphi_{11} = 0.4$)											
-0.8	0.5929	1.0702	0.7390	5.0357	3.6707	3.3831	0.2665	0.6255	0.4937	2.7205	3.0553	5.3516
-0.4	0.2920	0.6953	0.5228	2.1158	2.2366	2.8554	0.2149	0.3754	0.3121	1.7442	2.2647	3.0596
0.0	0.2768	0.6542	0.4895	1.8936	2.1101	2.7331	0.2114	0.3316	0.2901	1.6012	2.1221	2.5243
0.4	0.3114	0.7041	0.5475	2.5738	2.5347	3.0350	0.2215	0.3853	0.3219	1.8580	2.3553	3.0313
0.8	0.6167	1.0291	0.7955	5.8459	4.2715	3.5106	0.3073	0.6649	0.4991	2.9097	3.0789	4.7178
σ_{21}	($\rho = \varphi_{11} = 0.4$)											
-0.8	0.3235	0.6980	0.5330	2.6040	2.4923	3.1698	0.2226	0.3885	0.3210	1.8658	2.3659	2.9999
-0.4	0.3137	0.7253	0.5379	2.5161	2.5034	3.0659	0.2213	0.3814	0.3229	1.8313	2.3430	3.0091
0.0	0.3162	0.7335	0.5343	2.5589	2.5125	3.0405	0.2207	0.3789	0.3187	1.8383	2.3231	2.9954
0.4	0.3114	0.7041	0.5475	2.5738	2.5347	3.0350	0.2215	0.3853	0.3219	1.8580	2.3553	3.0313
0.8	0.3128	0.7202	0.5544	2.5216	2.5086	3.2253	0.2228	0.3794	0.3203	1.8627	2.3925	3.0314
φ_{11}	($\rho = \sigma_{21} = 0.4$)											
-0.8	0.3248	0.7585	0.5914	2.5320	2.5217	3.0695	0.2243	0.3779	0.3195	1.8703	2.3538	2.9324
-0.4	0.3237	0.7532	0.5547	2.5783	2.6128	3.1558	0.2239	0.3748	0.3181	1.8588	2.3488	2.9180
0.0	0.3248	0.7551	0.5414	2.5918	2.6151	3.1543	0.2236	0.3760	0.3176	1.8661	2.3451	2.9134
0.4	0.3114	0.7041	0.5475	2.5738	2.5347	3.0350	0.2215	0.3853	0.3219	1.8580	2.3553	3.0313
0.8	0.3577	0.8001	0.5825	2.5986	2.6180	3.2336	0.2226	0.3821	0.3242	1.8483	2.3537	3.0390

Panel D: $\beta(z) = 0.25\exp(-z^2)$

	$T = 100$						$T = 250$					
	<i>Restricted</i>			<i>Unrestricted</i>			<i>Restricted</i>			<i>Unrestricted</i>		
	LL			LL			LL			LL		
	SP	ROT	PLLR	SP	ROT	PLLR	SP	ROT	PLLR	SP	ROT	PLLR
ρ	($\sigma_{21} = \varphi_{11} = 0.4$)											
-0.8	0.2604	0.4373	0.3173	1.2857	1.0293	0.9385	0.0934	0.1890	0.1425	0.6156	0.7427	1.2842
-0.4	0.1298	0.2820	0.2329	0.6415	0.6669	0.8858	0.0678	0.1183	0.0990	0.5311	0.6950	0.8623
0.0	0.1232	0.2780	0.2170	0.6267	0.6562	0.8484	0.0638	0.1105	0.0920	0.5146	0.6820	0.8064
0.4	0.1381	0.2999	0.2306	0.7100	0.7397	0.9534	0.0663	0.1141	0.0956	0.5092	0.6450	0.8304
0.8	0.2679	0.4385	0.3477	1.5955	1.1008	0.9340	0.0843	0.2146	0.1551	0.7910	0.8805	1.4605
σ_{21}	($\rho = \varphi_{11} = 0.4$)											
-0.8	0.1397	0.2984	0.2270	0.6902	0.7542	0.9615	0.0667	0.1145	0.0953	0.5195	0.6480	0.8263
-0.4	0.1358	0.3142	0.2234	0.7124	0.7532	0.9088	0.0669	0.1137	0.0962	0.5167	0.6513	0.8262
0.0	0.1347	0.3020	0.2278	0.6698	0.7364	0.9120	0.0666	0.1125	0.0961	0.4952	0.6521	0.8666
0.4	0.1381	0.2999	0.2306	0.7100	0.7397	0.9534	0.0663	0.1141	0.0956	0.5092	0.6450	0.8304
0.8	0.1352	0.3220	0.2296	0.6860	0.7370	0.9395	0.0662	0.1128	0.0950	0.5110	0.6528	0.8379
φ_{11}	($\rho = \sigma_{21} = 0.4$)											
-0.8	0.1559	0.4078	0.3091	0.7239	0.7604	1.0141	0.0681	0.1222	0.1003	0.5332	0.6787	0.8455
-0.4	0.1283	0.3026	0.2422	0.6825	0.7350	0.9461	0.0655	0.1121	0.0927	0.5313	0.6704	0.8429
0.0	0.1285	0.2872	0.2205	0.6828	0.7377	0.9431	0.0650	0.1116	0.0917	0.5282	0.6703	0.8363
0.4	0.1381	0.2999	0.2306	0.7100	0.7397	0.9534	0.0663	0.1141	0.0956	0.5092	0.6450	0.8304
0.8	0.1818	0.3809	0.2967	0.7643	0.7804	0.9541	0.0692	0.1237	0.1025	0.5402	0.6881	0.8394