AMERICAN MATHEMATICAL SOCIETY
MathSciNet
Mathematical Reviews


| Citations | From References: 0 | From Reviews: 0 |
|---|---|---|

**MR3857826** 62G05 62G08
**Kroll, Martin** (D-MNHM-NDM)
**Non-parametric Poisson regression from independent and weakly dependent observations by model selection. (English summary)**
*J. Statist. Plann. Inference* **199** (2019), 249–270.

This article studies the problem of estimating a nonparametric Poisson regression model in which an integer-valued response $Y$ is the realization of a Poisson random variable with an unknown parameter $\lambda(X)$ depending on a covariate $X$. The $n$ observations of the covariate $X$ are assumed to be strictly stationary and either independent or weakly dependent. For the latter, $\beta$-mixing (or absolute regularity) is imposed.

Special attention is paid to the adaptive estimation in which the estimators of $\lambda$ are fully data-driven and thus free of structural assumptions on $\lambda$, and orthogonal projection estimation is examined throughout. Let $\mathcal{M}_n$ be a finite collection of models. For a given model $m \in \mathcal{M}_n$, let $\lambda_m$ be the orthogonal projection of $\lambda$, and denote the estimator for $\lambda_m$ by $\widehat{\lambda}_m$. Because each $\widehat{\lambda}_m$ depends on the model $m$, the optimal model $\widehat{m}$ is chosen from the set $\{\widehat{\lambda}_m\}_{m\in\mathcal{M}_n}$ via the penalized contrast criterion

$$\widehat{m} = \underset{m\in\mathcal{M}_n}{\arg\min} \left\{ \gamma_n\left(\widehat{\lambda}_m\right) + \widehat{\text{pen}}(m) \right\},$$

where $\gamma_n$ is the contrast function and $\widehat{\text{pen}}$ can be obtained by replacing the unobservable upper bound of $\lambda$ in the penalty term pen with its plug-in estimate.

It is demonstrated that for each of the cases of independent and weakly dependent observations, the optimal adaptive projection estimator $\widehat{\lambda}_{\widehat{m}}$ satisfies the oracle inequality

$$E\left\|\widehat{\lambda}_{\widehat{m}} - \lambda\right\|^2 \lesssim \min_{m\in\mathcal{M}_n} \max\left\{\|\lambda-\lambda_m\|^2, \text{pen}\,(m)\right\} + \frac{1}{n},$$

where the expression '$a_n \lesssim b_n$' means that $a_n \le Cb_n$ holds for all $n \in \mathbb{N}$ with some constant $C$ independent of $n$, and $\|\lambda-\lambda_m\|^2$ corresponds to the squared bias term due to the orthogonal projection $\lambda_m$. Concentration inequalities for Poisson point processes play a key role in the proof for each case. Notice that the order of magnitude in pen is the variance convergence rate multiplied by an additional logarithmic factor. It follows that $\widehat{\lambda}_{\widehat{m}}$ attains the minimax optimal rate up to the logarithmic factor regardless of whether independent or weakly dependent observations are used. *Masayuki Hirukawa*

© *Copyright American Mathematical Society 2019*