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Khismatullina, Marina (D-BONN-NDM);
Vogt, Michael [**Vogt, Michael**[2]] (D-BONN-EC)
Multiscale inference and long-run variance estimation in non-parametric regression with time series errors. (English summary)
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This article is concerned with a testing problem for certain shape properties of a nonparametric time trend. For T observations of a univariate time series $\{Y_{t,T}\}_{t=1}^T$, the test is built on the nonparametric regression model

$$Y_{t,T} = m\left(\frac{t}{T}\right) + \varepsilon_t,$$

where $m\colon [0,1] \to \mathbb{R}$ is an unknown nonparametric function, and $\{\varepsilon_t\}$ is a zero-mean stationary error process.

The null hypothesis for this test can be formulated as follows. Throughout it is assumed that m is continuously differentiable on $[0,1]$. Also let $H_0(u,h)$ be the hypothesis that m is constant on the interval $[u-h,u+h]$ for some location u and bandwidth h. It follows that $H_0(u,h) : m'(w) = 0$ for all $w \in [u-h,u+h]$, where m' is the first-order derivative of m. The overall null hypothesis considered in this article is designed to test for constancy of m on many such intervals simultaneously. It is in the form of

$$H_0 : \text{the hypothesis } H_0\,(u,h) \text{ holds true for all } (u,h) \in \mathcal{G}_T,$$

where $\mathcal{G}_T$ is some large set of points (u,h) that expands with the sample size T in general.

Accordingly, the test statistic for the overall null hypothesis H_0 is constructed from the one for $H_0(u,h)$. For the latter, the following rescaled local linear estimator of the derivative $m'(u)$ is considered:

$$\widehat{\psi}_T\,(u,h) = \sum_{t=1}^{T} w_{t,T}\,(u,h)\, Y_{t,T},$$

where

$$w_{t,T}\,(u,h) = \frac{\Lambda_{t,T}\,(u,h)}{\sqrt{\sum_{t=1}^T \Lambda_{t,T}^2\,(u,h)}},$$

$$\Lambda_{t,T}\,(u,h) = K\left(\frac{t/T-u}{h}\right)\left\{S_{T,0}\,(u,h)\left(\frac{t/T-u}{h}\right) - S_{T,1}\,(u,h)\right\},$$

$$S_{T,\ell}\,(u,h) = \frac{1}{Th}\sum_{t=1}^{T} K\left(\frac{t/T-u}{h}\right)\left(\frac{t/T-u}{h}\right)^{\ell}$$

for $\ell = 0,1,2,$ and K is a nonnegative symmetric kernel function with support on $[-1,1]$. The test statistic for $H_0(u,h)$ is then given by $\widehat{\psi}_T(u,h)/\widehat{\sigma}$, where $\widehat{\sigma}^2$ is some consistent estimator of the long-run variance of $\sigma^2 = \sum_{j=-\infty}^{\infty} E(\varepsilon_t\varepsilon_{t-j})$ of the error process $\{\varepsilon_t\}$. In particular, instead of leaving the dependent structure of the process unspecified, this article assumes that it obeys the class of AR(∞) processes and proposes a difference-based long-run variance estimator.

In the end, the test statistic for H_0 is defined as

$$\widehat{\Psi}_T = \max_{(u,h)\in\mathcal{G}_T} \left\{ \left| \frac{\widehat{\psi}_T(u,h)}{\widehat{\sigma}} \right| - \lambda(h) \right\},$$

where $\lambda(h) = \sqrt{2\log\{1/(2h)\}}$ is the additive correction term that is implied by the distribution of the maximum of standard normal random variables. Because the null distribution of this test statistic is non-standard, critical values should be computed by Monte Carlo simulation. Specifically, $q_T(\alpha)$, the critical value for a given level of significance $\alpha \in (0,1)$, can be obtained as the $(1-\alpha)$-quantile of the empirical distribution of

$$\Phi_T = \max_{(u,h)\in\mathcal{G}_T} \left\{ \left| \frac{\phi_T(u,h)}{\sigma} \right| - \lambda(h) \right\},$$

where $\phi_T(u,h) = \sum_{t=1}^{T} w_{t,T}(u,h)\sigma Z_t$ for $\{Z_t\}_{t=1}^{T} \stackrel{iid}{\sim} N(0,1)$. For such $q_T(\alpha)$, we reject the overall null hypothesis H_0 if $\widehat{\Psi}_T > q_T(\alpha)$. It is also demonstrated that the test has asymptotic power 1 against some local alternatives. *Masayuki Hirukawa*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.