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Nonparametric estimation in a regression model with additive and multiplicative noise. (English summary)

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A nonparametric regression model

$$Y = f(\mathbf{X})U + V$$

is investigated, where $f\colon [0,1]^d \to \mathbb{R}$ is unknown and (U, V) are multiplicative and additive errors so that U is independent of $\mathbf{X}$ and V. Applications of this model can be found, for instance, in stochastic frontier models and signal and image processing. The focus of this article is on wavelet estimation of the quadratic functional $r := f^2$ using n random samples $(Y_1, \mathbf{X}_1), \ldots, (Y_n, \mathbf{X}_n)$. Convergence rates of two estimators of r, namely, the linear $(\hat{r}_n^{\text{lin}})$ and nonlinear $(\hat{r}_n^{\text{non}})$ wavelet estimators, over the mean integrated squared error are derived. It is demonstrated that if r belongs to the Besov space $B^s_{p,q}([0,1]^d)$ with $p, q \in [1, \infty)$ and $s > d/p$ and other suitable regularity conditions hold, then

$$E\left[\int_{[0,1]^d} \left\{\hat{r}_n^{\text{lin}}(\mathbf{x}) - r(x)\right\}^2 d\mathbf{x}\right] \leq c n^{-\frac{2s'}{2s'+d}}, \text{ and}$$

$$E\left[\int_{[0,1]^d} \left\{\hat{r}_n^{\text{non}}(\mathbf{x}) - r(\mathbf{x})\right\}^2 d\mathbf{x}\right] \leq c(\ln n) n^{-\frac{2s}{2s+d}}$$

where $c > 0$ is a generic constant and $s' = s - d(1/p - 1/2)_+$ with $x_+ := \max\{x, 0\}$.

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.