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Goodness-of-fit tests for parametric specifications of conditionally heteroscedastic models. (English summary)

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This article studies a goodness-of-fit test for the conditional variance of a strictly stationary, multiplicative error process $x_t = \sigma_t \varepsilon_t$, where the innovation ε_t is an i.i.d. process with zero mean and unit variance that is independent of σ_t. Given that $\mathcal{I}_{t-1}$ is the information available at time t, the conditional variance of x_t can be expressed as $\sigma_t^2 = \mathbb{E}(x_t^2 | \mathcal{I}_{t-1})$. The null hypothesis considered in this article is

$$H_0 : \sigma_t^2 = h_{\boldsymbol{\vartheta}}(\boldsymbol{\Upsilon}_{t,p,q}) \quad \text{for some } \boldsymbol{\vartheta} \in \Theta \subseteq \mathbb{R}^v,$$

where $\boldsymbol{\Upsilon}_{t,p,q} = (x_{t-1}, \dots, x_{t-p}, \log(\sigma_{t-1}^2), \dots, \log(\sigma_{t-q}^2))'$, and $h_{\boldsymbol{\vartheta}}(\cdot)$ is a specific function depending on a v-dimensional parameter ϑ. The parametric specification $h_{\boldsymbol{\vartheta}}(\cdot)$ can cover a wide variety of GARCH models. After a suitable modification, this testing procedure can also be applied to autoregressive conditional duration models.

Observe that $\mathbb{E}(x_t^2 | \boldsymbol{\Upsilon}_{t,p,q}) = \sigma_t^2(\boldsymbol{\vartheta})$ under H_0. Let $\widehat{\boldsymbol{\vartheta}}_T$ be a consistent estimator of $\boldsymbol{\vartheta}$ (e.g., the quasimaximum likelihood estimator) using T observations. Also, given predicted values $\tilde{\sigma}_t(\widehat{\boldsymbol{\vartheta}}_T)$ based on some initial condition, denote standardized residuals as $\tilde{\varepsilon}_t = x_t / \tilde{\sigma}_t(\widehat{\boldsymbol{\vartheta}}_T)$. Then, the normalized integrated process

$$Q_{T,w} = (T-m) \int |S_T(\mathbf{u})|^2 w(\mathbf{u})\, d\mathbf{u}$$

is considered as the test statistic, where

$$S_T(\mathbf{u}) = \frac{1}{T-m} \sum_{t=m+1}^{T} (\tilde{\varepsilon}_t^2 - 1) \exp(i \mathbf{u}' \tilde{\boldsymbol{\Upsilon}}_t),$$

$m = \max\{p, q\}$, $i = \sqrt{-1}$, $\tilde{\boldsymbol{\Upsilon}}_t = (x_{t-1}, \dots, x_{t-p}, \log\{\tilde{\sigma}_{t-1}^2(\widehat{\boldsymbol{\vartheta}}_T)\}, \dots, \log\{\tilde{\sigma}_{t-q}^2(\widehat{\boldsymbol{\vartheta}}_T)\})'$, and $w(\mathbf{u})$ is some weight function. In practice, the Gaussian density is chosen as the weight function because of a theoretical requirement.

In essence, a large value of $Q_{T,w}$ leads to rejection of H_0. However, the asymptotic null distribution of $Q_{T,w}$ depends on the unknown distribution of ε_t. Then, the weighted bootstrap (also known as the multipliers method) is applied to approximate the null distribution. Let $\xi_{m+1}, \dots, \xi_T$ be i.i.d. copies of a random variable ξ with zero mean and unit variance that are independent of $x_1, \dots, x_T$. Then, as a fully operational counterpart of $Q_{T,w}$, consider

$$Q_{T,w}^* = \int |W_{1,T}^*(\mathbf{u}; \widehat{\boldsymbol{\vartheta}}_T)|^2 w(\mathbf{u})\, d\mathbf{u},$$

where

$$\begin{aligned}
W^{*}_{1,T}(\mathbf{u};\widehat{\boldsymbol{\vartheta}}_T) &= \frac{1}{\sqrt{T-m}}\sum_{t=m+1}^{T}\tilde{H}_t(\mathbf{u};\widehat{\boldsymbol{\vartheta}}_T)\xi_t,\\
\tilde{H}_t(\mathbf{u};\widehat{\boldsymbol{\vartheta}}_T) &= (\tilde{\varepsilon}_t^2-1)\{\cos(\mathbf{u}'\tilde{\boldsymbol{\Upsilon}}_t)+\sin(\mathbf{u}'\tilde{\boldsymbol{\Upsilon}}_t)\} - \tilde{G}(\mathbf{u};\widehat{\boldsymbol{\vartheta}}_T)'\widehat{L}_t,\\
\tilde{G}(\mathbf{u};\widehat{\boldsymbol{\vartheta}}_T) &= \frac{1}{T-m}\sum_{t=m+1}^{T}\tilde{A}_t(\widehat{\boldsymbol{\vartheta}}_T)\{\cos(\mathbf{u}'\tilde{\boldsymbol{\Upsilon}}_t)+\sin(\mathbf{u}'\tilde{\boldsymbol{\Upsilon}}_t)\},\\
\tilde{A}_t(\vartheta) &= \frac{\partial}{\partial\vartheta}\log\{\tilde{\sigma}_t^2(\vartheta)\},\\
\widehat{L}_t &= (\tilde{\varepsilon}_t^2-1)\tilde{A}_t(\widehat{\boldsymbol{\vartheta}}_T)\widehat{J}^{-1}, \text{ and}\\
\widehat{J} &= \frac{1}{T}\sum_{t=1}^{T}\tilde{A}_t(\widehat{\boldsymbol{\vartheta}}_T)\tilde{A}_t(\widehat{\boldsymbol{\vartheta}}_T)'.
\end{aligned}$$

It is demonstrated that both $Q_{T,w}$ and $Q^{*}_{T,w}$ have the same limiting conditional distribution, given the observations. The weighted bootstrap p-value can be approximated by simulating bootstrap samples of $Q^{*}_{T,w}$ many times. Observe that it suffices to estimate the parameter $\boldsymbol{\vartheta}$ only once in the weighted bootstrap. This is in sharp contrast to the residual-based bootstrap, which requires the parameter estimation for each resample.

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