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On linearization of nonparametric deconvolution estimators for repeated measurements model. (English summary)
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This article is concerned with nonparametric density deconvolution estimation for the classical measurement error model. Let a scalar variable Y be generated by $Y = X + \epsilon$, where X is the error-free latent variable having an unknown density f_X, ϵ is the measurement error with a density f_ϵ and $E(\epsilon) = 0$, and X and ϵ are independent. Suppose that we are primarily interested in estimating f_X nonparametrically, given the noisy measurement Y. The deconvolution problem has been typically studied either under the assumption that the distribution of ϵ is known or under the one that prior information about its shape is available (e.g., symmetry) even though the distribution is unknown.

In contrast, leaving both f_X and f_ϵ unspecified, T. Li and Q. H. Vuong [J. Multivariate Anal. **65** (1998), no. 2, 139–165; MR1625869] demonstrated that these densities can be estimated nonparametrically via repeated measurements. For n pairs of i.i.d. noisy measurements $\{(Y_{1j}, Y_{2j})\}_{j=1}^n = \{(X_j + \epsilon_{1j}, X_j + \epsilon_{2j})\}_{j=1}^n$, define the empirical characteristic function of (Y_1, Y_2) and its derivative with respect to the first argument as

$$\widehat{\psi}(u_1, u_2) = \frac{1}{n}\sum_{j=1}^n \exp\{i(u_1Y_{1j} + u_2Y_{2j})\} \quad \text{and}$$
$$\widehat{\psi}_1(u_1, u_2) = \frac{\partial\widehat{\psi}(u_1, u_2)}{\partial u_1} = \frac{1}{n}\sum_{j=1}^n iY_{1j}\exp\{i(u_1Y_{1j} + u_2Y_{2j})\},$$

respectively, where $i = \sqrt{-1}$. Then, relying on I. I. Kotlarski's [Pacific J. Math. **20** (1967), 69–76; MR0203769] identity, Li and Vuong [op. cit.] proposed to estimate the characteristic functions of X and ϵ as

$$\widehat{\varphi}_X(u) = \exp\int_0^u \frac{\widehat{\psi}_1(0, u_2)}{\widehat{\psi}(0, u_2)}du_2 \quad \text{and} \quad \widehat{\varphi}_\epsilon(u) = \frac{\widehat{\psi}(0, u)}{\widehat{\varphi}_X(u)},$$

respectively. Finally, two densities f_X and f_ϵ can be estimated by

$$\widehat{f}_a(t) = \frac{1}{2\pi}\int_{\mathbb{R}} e^{-iut}\widehat{\varphi}_a(u)\widehat{\varphi}_K(hu)du$$

for $a \in \{X, \epsilon\}$, where $\widehat{\varphi}_K(u) = \int_{\mathbb{R}} e^{iux}K(x)dx$ is the Fourier transform of a kernel function K, and h is the bandwidth.

In this article, it is demonstrated that $\widehat{f}_X - f_X$ and $\widehat{f}_\epsilon - f_\epsilon$ can be represented in an asymptotic linear form uniformly over a given compact interval. Four results are available, depending on whether the distributions of X and ϵ may be ordinary smooth or supersmooth. These results are obtained via intermediate Gaussian approximations. An advantage of utilizing such approximations is faster uniform convergence rates of $\widehat{f}_X$ and $\widehat{f}_\epsilon$ than not only those derived by Li and Vuong [op. cit.] but also those obtained in yet another article by the authors [Econometric Theory **38** (2022), no. 1, 172–193, doi:10.1017/S0266466620000572] via maximal inequalities. On the other hand, approximation errors in the sample counterparts of linearization terms have slow convergence rates. Therefore, the authors recommend using subsample-based bootstrap counterparts of the linearization terms to construct bootstrap confidence bands for $\widehat{f}_X$ and $\widehat{f}_\epsilon$.

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