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Nonlinear wavelet-based estimation to spectral density for stationary non-Gaussian linear processes. (English summary)

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Let $\{X_t\}_{t=1}^n$ be a realization of a stationary linear process $X_t=\sum_{j=0}^{\infty}\varphi_j Z_{t-j}$, where the innovation $\{Z_t\}$ is an i.i.d. sub-Gaussian process with zero mean and unit variance. In this paper, convergence rates of the mean integrated squared errors (MISEs) for two wavelet-based spectral estimators are derived.

Let $f(\omega)$, $\omega\in[-\pi,\pi]$, be the spectral density of $\{X_t\}$. The first estimator for $f(\omega)$ is

$$\overline{f}(\omega)=\frac{\widehat{\gamma}(0)}{\sqrt{2\pi}}+\sum_{j=0}^{j_0-1}\sum_{k=0}^{2^j-1}\widehat{\beta}_{jk}\Psi_{jk}(\omega)+\sum_{j=j_0}^{j_1}\sum_{k=0}^{2^j-1}\widehat{\beta}_{jk}\mathbf{1}\left\{\left|\widehat{\beta}_{jk}\right|>\delta_{jk}\right\}\Psi_{jk}(\omega),$$

where $\widehat{\gamma}(h)=(1/n)\sum_{t=|h|+1}^{n}(X_t-\overline{X})(X_{t-|h|}-\overline{X})$ with $\overline{X}=(1/n)\sum_{t=1}^{n}X_t$ being the sample autocovariance of $\{X_t\}$, $\{\Psi_{jk}(\cdot)\}$ is an orthonormal 2π-periodic wavelet basis, the $\widehat{\beta}_{jk}$ are empirical wavelet coefficients, and δ_{jk} is a thresholding constant. Notice that (j_0,j_1) are set to diverge to infinity more slowly than the sample size n. The estimator $\overline{f}(\omega)$ is infeasible because the thresholding constant δ_{jk} depends on unknown quantities. Then, a fully operational estimator $\widehat{f}(\omega)$ with δ_{jk} replaced by its empirical counterpart is also proposed as the second estimator.

Convergence properties of similar wavelet estimators have been investigated in the literature [e.g., M. H. Neumann, J. Time Ser. Anal. **17** (1996), no. 6, 601–633; MR1424908; P. Fryzlewicz, G. P. Nason and R. von Sachs, J. Time Ser. Anal. **29** (2008), no. 5, 868–880; MR2450900]. What distinguishes this article from others is the application of the Hanson-Wright inequality [e.g., M. Rudelson and R. Vershynin, Electron. Commun. Probab. **18** (2013), no. 82 (Theorem 1.1); MR3125258] to evaluate the convergence rates of empirical wavelet coefficients $\widehat{\beta}_{jk}$. This inequality is a general concentration one for quadratic forms of sub-Gaussian random variables, and the fact that the $\widehat{\beta}_{jk}$ are quadratic functions of $\{X_t\}$ establishes their convergence rates.

The main result of this article is

$$\sup_{f\in B^s_{p,q}(M)}E\left[\int_{-\pi}^{\pi}\left\{\check{f}(\omega)-f(\omega)\right\}^2d\omega\right]=O\left\{\left(\frac{\ln n}{n}\right)^{2s/(2s+1)}\right\}$$

for $\check{f}\in\{\overline{f},\widehat{f}\}$, where $B^s_{p,q}(M)$ is a Besov ball of radius $M>0$ with $s>0$, $p,q\in[1,\infty)$. The rate $(\ln n/n)^{2s/(2s+1)}$ in MISE is nearly optimal within a logarithmic term, compared with the MISE-optimal rate $n^{-2s/(2s+1)}$ of standard nonparametric regression estimators.

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.