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Estimation of functional-coefficient autoregressive models with measurement error. **(English summary)**
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Consider the time series $\{Y_t, t = 1, \ldots, n\}$ generated by the p-th order functional-coefficient autoregressive (FAR) model

$$Y_t = \sum_{j=1}^{p} \alpha_j(U_t) Y_{t-j} + \varepsilon_t.$$

It is often the case that time series data can be observed with measurement error. In light of this, this article is concerned with estimation of the FAR model when the true time series $\{Y_t, t = 1, \ldots, n\}$ is unobservable but the surrogate time series $\{Z_t, t = 1, \ldots, n\}$ is observable. It is assumed that the relation between Y_t and Z_t can be expressed as the classical measurement error model

$$Z_t = Y_t + e_t, \quad t = 1, \ldots, n,$$

where the measurement error e_t is independent of Y_t, while the covariate U_t that determines the coefficients $\alpha_1(\cdot), \ldots, \alpha_p(\cdot)$ is assumed to be free of measurement error.

The estimation theory for the $\alpha_j(\cdot)$ can be viewed as a FAR analog to those demonstrated by J. Staudenmayer and J. P. Buonaccorsi [J. Amer. Statist. Assoc. **100** (2005), no. 471, 841–852; MR2201013] for AR(p) models in the presence of measurement error. Local linear regression smoothing is adopted as the basic estimation strategy for the vector of functional coefficients $\boldsymbol{\alpha}_0 = (\alpha_1(u_0), \ldots, \alpha_p(u_0))^\top$ at a design point u_0. However, naive local linear estimation using the mismeasured Z_t in place of Y_t is inconsistent, due to the non-vanishing bias term that depends on the measurement error variance $\sigma_e^2 := \mathbb{E}(e_t^2)$, as in [op. cit.].

As long as σ_e^2 is known, the bias term is consistently estimable and thus a consistent, bias-corrected estimator for $\boldsymbol{\alpha}_0$ can be obtained. The bias-corrected estimator $\widehat{\mathbf{a}}^b = \widehat{\boldsymbol{\alpha}}(u_0)$ takes the form

$$\widehat{\mathbf{a}}^b = \mathbf{H}\left\{\frac{1}{n}\left(\mathbf{D}_u^Z\right)^\top \mathbf{W}\mathbf{D}_u^Z - \boldsymbol{\Omega}\right\}^{-1} \frac{1}{n}\left(\mathbf{D}_u^Z\right)^\top \mathbf{W}\mathbf{Z},$$

where $\mathbf{H} := [\mathbf{I}_p, \mathbf{0}_{p\times p}]$, $\mathbf{W} := \mathrm{diag}\{K_h(U_1 - u_0), \ldots, K_h(U_n - u_0)\}$, $K_h(\cdot) := K(\cdot/h)/h$ for a kernel function $K(\cdot)$ and a bandwidth h, $\widetilde{\mathbf{X}}_t := (Z_{t-1}, \ldots, Z_{t-p})^\top$, $\mathbf{Z} := (Z_1, \ldots, Z_n)^\top$,

$$\mathbf{D}_u^Z := \begin{bmatrix} \widetilde{\mathbf{X}}_1^\top & \widetilde{\mathbf{X}}_1^\top (U_1 - u_0)/h \\ \vdots & \vdots \\ \widetilde{\mathbf{X}}_n^\top & \widetilde{\mathbf{X}}_n^\top (U_n - u_0)/h \end{bmatrix}, \quad \boldsymbol{\Omega} := \sigma_e^2 \mathbf{I}_p \otimes \widehat{f}_U(u_0) \begin{bmatrix} 1 & \mu_1 \\ \mu_1 & \mu_2 \end{bmatrix},$$

$\widehat{f}_U(u_0) := (1/n)\sum_{t=1}^n K_w(U_t - u_0)$ is a kernel density estimator for the stationary distribution of U_t using yet another bandwidth w, and $\mu_k := \int u^k K(u)du$. Notice that the term $(1/n)(\mathbf{D}_u^Z)^\top \mathbf{W}\mathbf{D}_u^Z - \boldsymbol{\Omega}$ may be nonpositive definite in finite samples. Because $\widehat{\mathbf{a}}^b$ is not well-defined in this case, a finite sample correction of the term is also considered.

Asymptotic normality and weak uniform consistency with a rate of the estimator

$\widehat{\mathbf{a}}^b$ are established under some regularity conditions, including strict stationarity and strong mixing of $(U_t, \mathbf{X}_t, Y_t)$ for $\mathbf{X}_t = (Y_{t-1}, \ldots, Y_{t-p})^\top$. A consistent estimator of the regression error variance σ_ε^2 is also proposed. *Masayuki Hirukawa*

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