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Deconvolution of spherical data corrupted with unknown noise. (English. English summary)

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In this article, the authors study a deconvolution problem of multivariate spherical data. Suppose that an observation Y is contaminated in such a way that $Y = X + \epsilon$, where the signal X is a random variable that takes values on a $(d-1)$-dimensional sphere (for $d \geq 2$) with an unknown center and an unknown radius, and the noise $\epsilon \in \mathbb{R}^d$ is another random variable that is independent of the signal. Both signal and noise distributions are unknown, and the only condition imposed is that d coordinates of the noise are independently distributed.

As a natural extension of the results in [É. Gassiat, S. Le Corff and L. Lehéricy, Ann. Statist. **50** (2022), no. 1, 303–323; MR4382018], the authors develop estimation methods of the center, radius and density of the signal on the sphere using n iid observations $\{Y_i\}_{i=1}^n$, with the noise distribution left unspecified. In the first step of their analysis, it is demonstrated that the center, radius and density of the signal on the sphere are identifiable if the noise distribution is centered and has a finite first moment.

Secondly, estimators of the radius and density are defined as the joint minimizer of a contrast function on characteristic functions, and an estimator of the center is proposed subsequently. These estimators are shown to be consistent. In particular, the radius estimator is proven to have an almost parametric rate of convergence when the noise distribution has a finite second moment.

Finally, for $d = 2$ (i.e., in the case of circular signals), the authors establish that the center can be estimated at a nearly parametric rate, and that the signal density on the circle can be estimated at the same minimax rate as derived in [A. Goldenshluger, J. Multivariate Anal. **81** (2002), no. 2, 360–375; MR1906385], where it was assumed that the noise distribution is known. As in the papers cited above, it is confirmed that the convergence rate of the signal density does not depend on the tail decay rate of the Fourier transform of the error distribution, unlike standard deconvolution problems on $\mathbb{R}^d$.

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.