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High-dimensional cointegration and Kuramoto inspired systems. (English. English summary)

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In this article, the authors examine via simulations estimators of the coefficient matrix and its rank in a linear high-dimensional cointegration system. This work can be viewed as a natural extension of the authors' previous simulation study in a low-dimensional setup [J. Math. Biol. **75** (2017), no. 4, 845–883; MR3687222].

Here is the cointegrating system considered in the simulation study. Suppose that a $p(\gg 0)$-dimensional $I(1)$ process $\mathbf{y}_n = (y_{1n}, \dots, y_{pn})^\top \in \mathbb{R}^p$ has $r\,(< p)$ linear cointegrating relations. When there is no deterministic term in $\mathbf{y}_n$, this relation can be expressed as the following vector error correction model:

$$\Delta \mathbf{y}_n = \mathbf{\Pi} \mathbf{y}_{n-1} + \epsilon_n,$$

where $\Delta \mathbf{y}_n = \mathbf{y}_n - \mathbf{y}_{n-1}$, and the coefficient matrix $\mathbf{\Pi} \in \mathbb{R}^{p\times p}$ is of rank r. The white noise process $\epsilon_n \in \mathbb{R}^p$ is assumed to be Gaussian. Inspired by a linearized version of Kuramoto systems, the authors further impose a special network structure on the coefficient matrix $\mathbf{\Pi}$. Let $\mathbf{\Pi}$ be block-diagonal so that $\mathbf{\Pi} = \mathrm{diag}\{\mathbf{\Pi}_1, \dots, \mathbf{\Pi}_k\}$, where the sub-matrix $\mathbf{\Pi}_i \in \mathbb{R}^{p_i \times p_i}$ with $\sum_{i=1}^k p_i = p$ is symmetric and of rank $r_i = p_i - 1$. It follows that $r = \mathrm{rank}(\mathbf{\Pi}) = \sum_{i=1}^k r_i = p - k$.

Estimating the above system takes two steps. First, the rank r is determined by a sequential testing procedure based on the likelihood ratio statistic. Because the asymptotic null distribution of the test statistic is nonstandard, the authors rely on a bootstrapping technique to obtain the rank estimate $\widehat{r}$. Given $\widehat{r}$, in the second step, the matrix $\mathbf{\Pi}$ is estimated. A few different estimators of $\mathbf{\Pi}$ are considered in the simulation study, and some of them are indeed subject to restrictions of symmetry and low rank.

There are two important findings from the simulation study. First, the sequential testing procedure tends to underestimate the rank r. However, graphical inspections indicate that precise estimation of r is not crucially important, as long as the degree of underestimation is relatively small. Second, given the rank estimate $\widehat{r}$ (which is slightly smaller than the true r), the symmetrized version of the ordinary least squares estimator of $\mathbf{\Pi}$ under a low rank approximation outperforms other competing estimators.

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.