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Quasi-maximum likelihood estimation of long-memory linear processes. (English. English summary)

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This article explores convergence properties of the quasi-maximum likelihood estimator (QMLE) for long-memory linear processes. The estimation theory starts from establishing that a long-memory one-sided linear process $(X_t)_{t\in\mathbb{Z}}$ can be represented as a long-memory AR(∞) process and vice versa. More precisely, based on this correspondence, $(X_t)_{t\in\mathbb{Z}}$ alternatively takes the form of

$$X_t = \sum_{k=1}^{\infty} u_k(\theta) X_{t-k} + \sigma\varepsilon_t,$$

where $(\varepsilon_t)_{t\in\mathbb{Z}}$ is a centered independent random variable (or a strong white noise) with a unit variance, $\theta = (\gamma, \sigma^2)^\top \in \Theta$ is the set of model parameters on a compact parameter space $\Theta \subseteq \mathbb{R}^{p-1} \times (0, \infty)$, and $(u_n(\theta))_{n\in\mathbb{N}}$ is a sequence of real numbers satisfying, for any $\theta \in \Theta$, $u_n(\theta) = L_\theta(n) n^{-d(\theta)-1}$ and $\sum_{n=1}^{\infty} u_n(\theta)p = 1$ for some slowly varying function $L_\theta(\cdot)$ and the memory parameter $d(\theta) \in (0, 1/2)$. Throughout it is assumed that $(u_n(\theta))$ does not depend on σ^2.

The AR(∞) representation of $(\varepsilon_t)_{t\in\mathbb{Z}}$ allows for the Gaussian QML estimation of θ. The QMLE for θ, denoted as $\widehat{\theta}_n = (\widehat{\gamma}_n, \widehat{\sigma}^2_n)^\top$ hereafter, is defined as

$$\widehat{\theta}_n = \arg\max_{\theta\in\Theta} \widehat{I}_n(\theta) := \arg\max_{\theta\in\Theta} -\frac{1}{2}\sum_{t=1}^{n}\left(\log(\sigma^2) + \frac{(X_t - \widehat{m}_t(\theta))^2}{\sigma^2}\right),$$

where $\widehat{m}_t(\theta) := \sum_{k=1}^{t-1} u_k(\theta) X_{t-k}$. Since $(u_n(\theta))$ is independent of σ^2, the QMLE reduces to

$$\widehat{\gamma}_n = \arg\min_{\gamma} \sum_{t=1}^{n}\left(X_t - \sum_{k=1}^{t-1} u_k(\gamma) X_{t-k}\right)^2$$

and

$$\widehat{\sigma}^2_n = \frac{1}{n}\sum_{t=1}^{n}\left(X_t - \sum_{k=1}^{t-1} u_k(\widehat{\gamma}_n) X_{t-k}\right)^2.$$

Observe that $\widehat{\gamma}_n$ is also a nonlinear least-squares estimator of γ.

It is demonstrated that under some regularity conditions, $\widehat{\theta}_n$ converges almost surely to θ and is asymptotically normal with the parametric rate of convergence. Y. Boubacar Maïnassara, Y. Esstafa, and B. Saussereau [Stat. Inference Stoch. Process. **24** (2021), no. 3, 549–608; MR4321851] already established almost sure convergence and asymptotic normality of the QMLE for FARIMA processes with a weak white noise. Convergence results in this article can be viewed as a generalization of [op. cit.] to all long-memory one-sided linear processes.

Moreover, the asymptotic distribution of the QMLE in this article is identical to the one for the Whittle estimator by L. Giraitis and D. Surgailis [Probab. Theory Related Fields **86** (1990), no. 1, 87–104; MR1061950], and thus these estimators are first-order asymptotically equivalent. Monte Carlo simulations indicate that finite-sample properties of these estimators have no substantial difference for the cases of 1000 or more observations, as their asymptotic theories predict, whereas the QMLE performs better than the Whittle estimator for the case of 300 observations.

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.