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Nonparametric estimation of copulas and copula densities by orthogonal projections. (English. English summary)

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In this article, the authors propose a nonparametric copula density estimator for a d-dimensional random vector $\mathbf{X} \in \mathbb{R}^d$. The proposed estimator is based on shifted Legendre orthogonal polynomials, and W. Gui [*Adaptive series estimators for copula densities*, Ph.D. thesis, Florida State Univ., 2009; MR2714078] already studied an equivalent estimator in the bivariate case. Therefore, the estimator may be viewed as a natural extension of Gui's estimator to the general d-dimensional case.

The copula density estimator is characterized by copula coefficients, which are built on the square integrability condition for the underlying copula density. This condition is not restrictive in practice; while copulas exhibiting lower or upper tail dependence do not satisfy the condition, Monte Carlo results in this article indicate that the proposed estimator works well for such copulas. The copula coefficients can be also used as an indicator of independence across components of $\mathbf{X}$.

The original (or hypothetical) copula density estimator possibly contains infinitely many copula coefficients. Because this estimator is of less practical use, the authors consider its approximation. Let $\mathbf{N} = \mathbf{N}(n) := (N_1, \ldots, N_d) \in \mathbb{N}^d$ be the set of d numbers of copula coefficients, where n is the sample size and $N_j = N_j(n) \to \infty$ for $j \in \{1, \ldots, d\}$ as $n \to \infty$. The number $\mathbf{N}$ plays a role in controlling the bias-variance trade-off of the copula density estimator, as done by the bandwidth in kernel density estimation. It is demonstrated that under some regularity conditions, the $\mathbf{N}$-th order approximation to the original copula density estimator is consistent in the mean integrated squared sense and asymptotically normal. To implement the approximated copula density estimator, the authors advocate employing the least-square cross-validation (LSCV) to select the number $\mathbf{N}$. The asymptotic optimality of LSCV is also established.

Extensive Monte Carlo simulations confirm superior finite-sample properties of the copula density estimator. The estimator has only one serious competitor—the local log-quadratic probit-transformation estimator by G. Geenens, A. Charpentier and D. Paindaveine [Bernoulli **23** (2017), no. 3, 1848–1873; MR3624880], although this estimator is defined only within the bivariate case. Two empirical examples using insurance and exchange rate data also illustrate practical usefulness of the proposed estimator.

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