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Corrigendum

Corrigendum to “Nonparametric multiplicative bias correction for kernel-type density estimation on the unit interval” [Comput. Statist. Data Anal. 54 (2010) 473–495]

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1. Introduction

In the article “Nonparametric multiplicative bias correction for kernel-type density estimation on the unit interval” by Masayuki Hirukawa, published in Volume 54 of *Computational Statistics & Data Analysis* (2010; pp. 473–495; “the Article” hereinafter), the author makes an error in deriving bias approximations of two TS-MBC estimators that are given in Theorem 1. Specifically, the constant right before $p_B(x)\, b^2$ in *bias* $\left\{\tilde{f}_{TS,B}(x)\right\}$ and the one right before $p_{MB}(x)\, b^2$, $p_{MB,0}(x)\, b^2$ and $p_{MB,1}(x)\, b^2$ in *bias* $\left\{\tilde{f}_{TS,MB}(x)\right\}$ are not $1/\{c(1-c)\}$ but $1/c$. This error is solely due to the author’s miscalculation. Eqs. (2)–(3) on p. 488 actually imply that

$$\frac{1}{1-c}\log I_b(x) - \frac{c}{1-c}\log I_{b/c}(x) = \log f(x) - \frac{1}{c}\left\{\frac{a_2(x)\, f(x) - \frac{1}{2}a_1^2(x)}{f^2(x)}\right\} b^2 + o\left(b^2\right),$$

and the bias approximations in Section 2 follow. The author thanks Dr. Nabil Zougab for pointing out the error in his personal communication and apologizes for any inconvenience that the error might have caused.

The remainder of this corrigendum is organized as follows. Section 2 itemizes corrections in the Article. Section 3 makes corrections in Hirukawa and Sakudo (2014), who also study TS-MBC estimation and commit the same error. Additionally, a supplementary material containing corrected simulation results for the Article is available on the author’s webpage.

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2. Corrections in the article

Theorem 1: Approximations to biases of two TS-MBC estimators should read

$$bias\left\{\tilde{f}_{TS,B}(x)\right\} \sim \frac{1}{c}p_B(x)b^2$$
$$= \frac{1}{c}\left[\frac{1}{2}\frac{\left\{(1-2x)f'(x)+\frac{1}{2}x(1-x)f''(x)\right\}^2}{f(x)} - \left\{-2(1-2x)f'(x)+\frac{1}{2}\left(11x^2-11x+2\right)f''(x)\right.\right.$$
$$\left.\left. + \frac{5}{6}x(1-x)(1-2x)f'''(x)+\frac{1}{8}x^2(1-x)^2f''''(x)\right\}\right]b^2, \text{ and}$$

$$bias\left\{\tilde{f}_{TS,MB}(x)\right\} \sim \frac{1}{c}\times\begin{cases} p_{MB}(x)b^2 & \text{for } x\in[2b,1-2b] \\ p_{MB,0}(x)b^2 & \text{for } x\in[0,2b) \\ p_{MB,1}(x)b^2 & \text{for } x\in(1-2b,1] \end{cases}$$

$$= \frac{1}{c}\times\begin{cases} \left[\frac{1}{8}\frac{x^2(1-x)^2\left\{f''(x)\right\}^2}{f(x)} - \left\{-\frac{1}{2}x(1-x)f''(x) + \frac{1}{3}x(1-x)(1-2x)f'''(x)+\frac{1}{8}x^2(1-x)^2f''''(x)\right\}\right]b^2 & \text{for } x\in[2b,1-2b] \\ \frac{1}{2}\left[\frac{\xi_b^2(x)\left\{f'(x)\right\}^2}{f(x)} - \left\{\xi_b^2(x)+\xi_b(x)+\frac{x}{b}\right\}f''(x)\right]b^2 & \text{for } x\in[0,2b) \\ \frac{1}{2}\left[\frac{\xi_b^2(1-x)\left\{f'(x)\right\}^2}{f(x)} - \left\{\xi_b^2(1-x)+\xi_b(1-x)+\frac{1-x}{b}\right\}f''(x)\right]b^2 & \text{for } x\in(1-2b,1]. \end{cases}$$

Mathematical expressions on pp. 477–479: Expressions of $MSE\left\{\tilde{f}_{TS,j}(x)\right\}$, $b^*_{TS,j}$, $\gamma(c)$, $MISE\left\{\tilde{f}_{TS,j}(x)\right\}$, $b^{**}_{TS,j}$, $E\left\{\int_0^1\tilde{f}_{TS,j}(x)\,dx\right\}$, $bias\left\{\tilde{f}^R_{TS,j}(x)\right\}$, and $\hat{b}_{TS.MB}$ should be corrected to

$$MSE\left\{\tilde{f}_{TS,j}(x)\right\} = \frac{p_j^2(x)}{c^2}b^4 + \frac{n^{-1}b^{-1/2}\lambda(c)f(x)}{2\sqrt{\pi}\sqrt{x(1-x)}} + o\left(b^4+n^{-1}b^{-1/2}\right),$$

$$b^*_{TS,j} = \left\{c^2\lambda(c)\right\}^{2/9}\left\{\frac{f(x)}{16\sqrt{\pi}\sqrt{x(1-x)}p_j^2(x)}\right\}^{2/9}n^{-2/9},$$

$$\gamma(c) = \left\{\frac{\left(1+c^{5/2}\right)(1+c)^{1/2}-2\sqrt{2}c^{3/2}}{c^{1/4}(1+c)^{1/2}(1-c)^2}\right\}^{8/9},$$

$$MISE\left\{\tilde{f}_{TS,j}(x)\right\} = \frac{b^4}{c^2}\int_0^1 p_j^2(x)\,dx + \frac{n^{-1}b^{-1/2}\lambda(c)}{2\sqrt{\pi}}\int_0^1\frac{f(x)}{\sqrt{x(1-x)}}dx + o\left(b^4+n^{-1}b^{-1/2}\right),$$

$$b^{**}_{TS,j} = \left\{c^2\lambda(c)\right\}^{2/9}\left\{\frac{\int_0^1 f(x)/\sqrt{x(1-x)}dx}{16\sqrt{\pi}\int_0^1 p_j^2(x)\,dx}\right\}^{2/9}n^{-2/9},$$

$$E\left\{\int_0^1\tilde{f}_{TS,j}(x)\,dx\right\} = 1+\frac{b^2}{c}\int_0^1 p_j(x)\,dx + o\left(b^2\right),$$

$$bias\left\{\tilde{f}^R_{TS,j}(x)\right\} \sim \frac{1}{c}\left\{p_j(x)-\int_0^1 p_j(x)\,dx\right\}b^2, \text{ and}$$

$$\hat{b}_{TS.MB} = \cdots = \arg\min_b \frac{b^4}{c^2}\int_0^1\tilde{p}^2_{MB}(x)w(x)\,dx + \frac{n^{-1}b^{-1/2}\lambda(c)}{2\sqrt{\pi}}\int_0^1\frac{g(x)}{\sqrt{x(1-x)}}w(x)\,dx,$$

respectively.

Beta-referenced smoothing parameter in Appendix A.3: The correct formula of $\hat{b}_{TS,MB}$ is given by

$$\hat{b}_{TS,MB} = \left\{c^2\lambda(c)\right\}^{2/9}\left\{\frac{B(\alpha,\beta)B(\alpha+9/2,\beta+9/2)}{16\sqrt{\pi}C_{TS,MB}(\alpha,\beta)}\right\}^{2/9}n^{-2/9},$$

where the expression of $C_{TS,MB}(\alpha,\beta)$ remains unchanged. Monte Carlo simulations using this formula are also conducted; see the supplementary material for details. Although the formula tends to yield a slightly larger smoothing parameter value than the one provided in the Article, the updated simulation results are qualitatively similar to those reported in the Article. In conclusion, effect of this correction on Monte Carlo results is minor at best.

3. Corrections in Hirukawa and Sakudo (2014)

Theorem 1: The bias approximation of the TS-MBC estimator should read

$$Bias\left\{\tilde{f}_{TS,j}(x)\right\} \sim \frac{1}{c}\left[\frac{1}{2}\left\{\frac{a_{1,j}^2(x,f)}{f(x)}\right\} - a_{2,j}(x,f)\right]b^2 := \frac{1}{c}p_j(x)\,b^2.$$

Mathematical expressions on pp. 115–117: Expressions of $MSE\left\{\tilde{f}_{TS,j}(x)\right\}$, $b_{TS,j}^{*}$, $MISE\left\{\tilde{f}_{TS,j}(x)\right\}$, $b_{TS,j}^{**}$, and $\hat{b}_{GR-TS}$ should be corrected to

$$MSE\left\{\tilde{f}_{TS,j}(x)\right\} = \frac{p_j^2(x)}{c^2}b^4 + n^{-1}b^{-1/2}\lambda(c)v_j(x)f(x) + o\left(b^4 + n^{-1}b^{-1/2}\right),$$

$$b_{TS,j}^{*} = \left\{c^2\lambda(c)\right\}^{2/9}\left\{\frac{v_j(x)f(x)}{8p_j^2(x)}\right\}^{2/9} n^{-2/9},$$

$$MISE\left\{\tilde{f}_{TS,j}(x)\right\} = \frac{b^4}{c^2}\int_0^\infty p_j^2(x)\,dx + \frac{\lambda(c)}{nb^{1/2}}\int_0^\infty v_j(x)f(x)\,dx + o\left(b^4 + n^{-1}b^{-1/2}\right),$$

$$b_{TS,j}^{**} = \left\{c^2\lambda(c)\right\}^{2/9}\left\{\frac{\int_0^\infty v_j(x)f(x)\,dx}{8\int_0^\infty p_j^2(x)\,dx}\right\}^{2/9} n^{-2/9}, \text{ and}$$

$$\hat{b}_{GR-TS} = \cdots = \arg\min_b\left\{\frac{b^4}{c^2}\int_0^\infty \tilde{p}_{MG}^2(x)\,w_{TS}(x)\,dx + \frac{\lambda(c)}{2\sqrt{\pi}nb^{1/2}}\int_0^\infty \frac{g(x)}{\sqrt{x}}w_{TS}(x)\,dx\right\},$$

respectively. Moreover, the expression of $\gamma(c)$ is exactly the same as the one provided in Section 2.

First equation in Appendix A: The first equation in Outline Proof of Theorem 1 should read

$$E\left\{\tilde{f}_{TS,G}(x)\right\} = f(x) + \frac{1}{c}\left[\frac{1}{2}\left\{\frac{a_{1,G}^2(x,f)}{f(x)}\right\} - a_{2,G}(x,f)\right]b^2 + o\left(b^2\right).$$

Gamma-referenced smoothing parameter in Appendix B: The correct formula of $\hat{b}_{GR-TS}$ is given by

$$\hat{b}_{GR-TS} = \left\{c^2\lambda(c)\right\}^{2/9}\left\{\frac{4^\alpha\beta^{9/2}\Gamma(\alpha+9/2)\,\Gamma(\alpha)}{16\sqrt{\pi}C_{TS}(\alpha)\,\Gamma(2\alpha)}\right\}^{2/9} n^{-2/9},$$

where the expression of $C_{TS}(\alpha)$ remains unchanged. Monte Carlo results based on this formula are available upon request. Again, influence of this correction on Monte Carlo results is minor.

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References

Hirukawa, M., Sakudo, M., 2014. Nonnegative bias reduction methods for density estimation using asymmetric kernels. Comput. Statist. Data Anal. 75, 112–123.