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REEXAMINATION OF THE ROBUSTNESS OF THE FAMA-FRENCH THREE-FACTOR MODEL

Masayuki Hirukawa[1] **and Jiro Hodoshima**[2,*]

[1]Faculty of Economics
Setsunan University
Neyagawa, Osaka 572-8508
Japan
e-mail: hirukawa@econ.setsunan.ac.jp

[2]Faculty of Economics
Nagoya University of Commerce and Business
Nisshin, Aichi 470-0193
Japan
e-mail: hodoshima@nucba.ac.jp

Abstract

We reexamine the robustness of the inference from the least-squares estimator under hetero-skedasticity and autocorrelation of unknown form in a generic multifactor asset pricing model. It is shown that the asymptotic covariance matrix of the least-squares estimator of betas depends only on the long-run cokurtosis of factors and error terms, whereas that of alphas depends not only on the long-run cokurtosis but also on the long-run coskewness of factors and error terms. We numerically evaluate the celebrated Fama-French three-factor model

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*Corresponding author

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using the U.S. data and find considerable changes in sizes of asymptotic variance estimates of the least-squares estimator of alphas and betas due to nonnormality and serial dependence.

1. Introduction

Estimation and testing of asset pricing models are fundamental in financial economics and financial econometrics. Questions of their empirical validity have created an enormous amount of research. One important direction of the study is robustness of the inference of asset pricing models against underlying assumptions made. Testing mean-variance efficiency of a given portfolio has been popular as testing Sharpe [12] and Lintner's [8] capital asset pricing model. Assuming normality on stock returns, Gibbons et al. [4] provided a well-known exact test. MacKinlay and Richardson [9] investigated the robustness of the test by studying the effect of nonnormality of the underlying distribution on the asymptotic distribution of the least-squares estimator ("LSE"), and propose an asymptotic test based on the asymptotic covariance matrix of the LSE under nonnormality, i.e., the generalized method of moments by Hansen [5]. Zhou [13] proposed another exact test assuming a class of elliptical distributions for the underlying distribution. Ando and Hodoshima [1] shown how nonnormality affects the inference of the LSE of alphas and betas by deriving the asymptotic covariance matrix formulas for the LSE of alphas and betas when the underlying data-generating process ("DGP") is independently and identically distributed ("i.i.d.") but not restricted to be normal. In these works, the main focus has been on whether the underlying distribution is normal or not while the i.i.d. assumption is maintained.

This note aims at studying the robustness of the inference based on the LSE in a generic multifactor asset pricing model under heteroskedasticity and autocorrelation of unknown form. Ando and Hodoshima [1] studied the robustness of the LSE of alphas and betas in the generic multifactor asset pricing model when factors and error terms are jointly i.i.d. with finite fourth moments and the joint distribution may not be normal. They find that the

asymptotic covariance matrix of the LSE of betas depends on the cokurtosis of factors and error terms, whereas that of alphas depends not only on the cokurtosis but also on the coskewness of factors and error terms. This implies that the asymptotic covariance matrix of the LSE of betas depends on the degree of tail-thickness of the underlying joint distribution but not on skewness measures of the distribution. In this note, we relax the i.i.d. assumption of the underlying joint distribution and investigate the asymptotic covariance matrix of the LSE of alphas and betas under heteroskedasticity and autocorrelation of unknown form. We demonstrate that while the asymptotic covariance matrix of the LSE of betas depends only on the long-run cokurtosis of factors and error terms, that of alphas depends not only on the long-run cokurtosis but also on the long-run coskewness of factors and error terms. In other words, the result of Ando and Hodoshima [1] under the i.i.d. nonnormal assumption is shown to continue to hold under the assumption of heteroskedasticity and autocorrelation of unknown form.

Obtaining the asymptotic covariance matrix of the LSE under heteroskedasticity and autocorrelation of unknown form is not new. However, to the best of our knowledge, the asymptotic covariance matrix for subsets of parameters in the generic multifactor asset pricing model has never been derived explicitly in the context of heteroskedasticity and autocorrelation of unknown form. Our asymptotic covariance matrix formulas of the LSE of alphas and betas in this framework are new and should be useful to reveal how nonnormality and serial dependence of the underlying joint distribution affect the inference from the LSE of alphas and betas.

Based on the analytical result we present, we reexamine the robustness of the Fama-French three-factor model under heteroskedasticity and autocorrelation of unknown form using the U.S. monthly and daily data. The model proposed by Fama and French [3] is quite popular and one of the benchmark asset pricing models. We find substantial effects of nonnormality and serial dependence on sizes of asymptotic variance estimates in the

Fama-French three-factor model, particularly in daily data. Typically, the asymptotic variance estimate becomes larger under heteroskedasticity and autocorrelation of unknown form than under the i.i.d. normal or nonnormal assumption.

This note is organized as follows: Section 2 derives the asymptotic covariance matrix of the LSE of alphas and betas in the multifactor asset pricing model. Section 3 reexamines the Fama-French three-factor model using the U.S. monthly and daily data when the asymptotic long-run covariance matrix derived in Section 2 is estimated by the method of heteroskedasticity and autocorrelation consistent ("HAC") covariance matrix estimation. Section 4 presents concluding comments. Appendix provides a set of regularity conditions for HAC estimation.

2. The Asymptotic Covariance Matrix of the LSE in the Multifactor Asset Pricing Model

2.1. The model

Let $\mathbf{R}_t \in \mathbb{R}^N$ and $\mathbf{f}_t \equiv (f_{1t}, \ldots, f_{Kt})' \in \mathbb{R}^K$ be vectors of N asset returns and K factors, respectively. Given T observations $\{(\mathbf{R}_t, \mathbf{f}_t)\}_{t=1}^T$, consider a multifactor asset pricing model

$$\mathbf{R}_t = \boldsymbol{\alpha} + \boldsymbol{\beta}_1 f_{1t} + \cdots + \boldsymbol{\beta}_K f_{Kt} + \boldsymbol{\varepsilon}_t, \qquad (1)$$

where parameter vectors $\boldsymbol{\alpha} \in \mathbb{R}^N$ and $\boldsymbol{\beta} \equiv (\boldsymbol{\beta}_1', \ldots, \boldsymbol{\beta}_K') \in \mathbb{R}^{NK}$ are referred to as "alphas" and "betas", and $\boldsymbol{\varepsilon}_t \in \mathbb{R}^N$ is the vector of error terms. For a more concise expression of (1), define the $N \times (K+1)$ parameter matrix $\boldsymbol{\Theta}$ as $\boldsymbol{\Theta} \equiv [\boldsymbol{\alpha}\ \boldsymbol{\beta}_1 \cdots \boldsymbol{\beta}_K]$, and write $\mathbf{X}_t \equiv (1, \mathbf{f}_t')'$. Then, equation (1) can be rewritten as

$$\mathbf{R}_t = \boldsymbol{\Theta}\mathbf{X}_t + \boldsymbol{\varepsilon}_t.$$

The error terms $\boldsymbol{\varepsilon}_t$ is assumed to have mean zero, i.e., $E(\boldsymbol{\varepsilon}_t) = 0$.

2.2. The asymptotic covariance matrix of the LSE of $\boldsymbol{\Theta}$ under heteroskedasticity and autocorrelation of unknown form

For the vector process $\mathbf{v}_t \equiv \mathrm{vec}(\boldsymbol{\varepsilon}_t\mathbf{X}_t')$, we assume that $E(\mathbf{v}_t) = \mathbf{0}$ holds, i.e., $\boldsymbol{\varepsilon}_t$ and $\mathbf{X}_t$ are uncorrelated. This assumption ensures the consistency of the LSE given by

$$\hat{\boldsymbol{\Theta}} = \sum_{t=1}^T \mathbf{R}_t\mathbf{X}_t'\left(\sum_{t=1}^T \mathbf{X}_t\mathbf{X}_t'\right)^{-1}.$$

The asymptotic distribution of $\sqrt{T}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}) \equiv \sqrt{T}\{\mathrm{vec}(\hat{\boldsymbol{\Theta}}) - \mathrm{vec}(\boldsymbol{\Theta})\}$ is also given by

$$\sqrt{T}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}) \xrightarrow{d} N(\mathbf{0}, \mathbf{V})$$

for some $N(K+1) \times N(K+1)$ asymptotic covariance matrix $\mathbf{V}$. When $\mathbf{v}_t$ has heteroskedasticity and autocorrelation of unknown form, $\mathbf{V}$ can be expressed as

$$\mathbf{V} \equiv \mathbf{H}^{-1}\mathbf{S}\mathbf{H}^{-1}, \qquad (2)$$

where

$$\mathbf{H} \equiv \mathbf{F} \otimes \mathbf{I}_N = E(\mathbf{X}_t\mathbf{X}_t') \otimes \mathbf{I}_N = \begin{bmatrix} 1 & E(\mathbf{f}_t') \\ E(\mathbf{f}_t) & E(\mathbf{f}_t\mathbf{f}_t') \end{bmatrix} \otimes \mathbf{I}_N,$$

with $\mathbf{F} \equiv E(\mathbf{X}_t\mathbf{X}_t')$, and $\mathbf{S}$ is the long-run covariance matrix ("LRCM") of the process $\mathbf{v}_t$ that takes the form of

$$\mathbf{S} \equiv \sum_{l=-\infty}^{\infty} \boldsymbol{\Gamma}_v(l) \equiv \sum_{l=-\infty}^{\infty} E(\mathbf{v}_t\mathbf{v}_{t-l}') = \sum_{l=-\infty}^{\infty} E(\mathbf{X}_t\mathbf{X}_{t-l}' \otimes \boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_{t-l}').$$

The final equality is established by recognizing $\mathbf{v}_t = \mathbf{X}_t \otimes \boldsymbol{\varepsilon}_t$. Observe that if $\mathbf{v}_t$ has no serial dependence, then $\mathbf{S}$ reduces to $\mathbf{S}_0 \equiv \boldsymbol{\Gamma}_v(0) = E(\mathbf{X}_t\mathbf{X}_t' \otimes \boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t')$ so that $\mathbf{V}$ is simplified as

$$\mathbf{V}_0 \equiv \mathbf{H}^{-1}\mathbf{S}_0\mathbf{H}^{-1}.$$

Moreover, as argued in Ando and Hodoshima [1], when the joint distribution of $(\boldsymbol{\varepsilon}_t', \mathbf{f}_t')'$ is i.i.d. normal, $\mathbf{V}$ collapses to

$$\mathbf{V}_G \equiv \mathbf{F}^{-1} \otimes E(\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t').$$

Notice that $\mathbf{F}$ can be rewritten as

$$\mathbf{F} = \begin{bmatrix} 1 & \boldsymbol{\mu}' \\ \boldsymbol{\mu} & \mathbf{V}_f + \boldsymbol{\mu}\boldsymbol{\mu}' \end{bmatrix},$$

where $\boldsymbol{\mu} \equiv E(\mathbf{f}_t)$ and $\mathbf{V}_f \equiv Var(\mathbf{f}_t)$ is the instantaneous covariance matrix of $\mathbf{f}_t$. Then, we have

$$\mathbf{F}^{-1} = \begin{bmatrix} 1 + \boldsymbol{\mu}'\mathbf{V}_f^{-1}\boldsymbol{\mu} & -\boldsymbol{\mu}'\mathbf{V}_f^{-1} \\ -\mathbf{V}_f^{-1}\boldsymbol{\mu} & \mathbf{V}_f^{-1} \end{bmatrix}.$$

When $\mathbf{v}_t$ has heteroskedasticity and autocorrelation of unknown form

$$\mathbf{V} = (\mathbf{F}^{-1} \otimes \mathbf{I}_N) \sum_{l=-\infty}^{\infty} E(\mathbf{X}_t\mathbf{X}_{t-l}' \otimes \boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_{t-l}')(\mathbf{F}^{-1} \otimes \mathbf{I}_N)$$

$$\equiv \sum_{l=-\infty}^{\infty} E(\mathbf{Y}_t\mathbf{Y}_{t-l}' \otimes \boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_{t-l}'), \tag{3}$$

where

$$\mathbf{Y}_t = \mathbf{F}^{-1}\mathbf{X}_t = \begin{bmatrix} 1 - \boldsymbol{\mu}'\mathbf{V}_f^{-1}(\mathbf{f}_t - \boldsymbol{\mu}) \\ \mathbf{V}_f^{-1}(\mathbf{f}_t - \boldsymbol{\mu}) \end{bmatrix}. \tag{4}$$

Now partition $\mathbf{V}$ as

$$\mathbf{V} = \begin{bmatrix} \mathbf{V}_{11} & \mathbf{V}_{12} \\ \mathbf{V}_{12}' & \mathbf{V}_{22} \end{bmatrix}.$$

Observe that $\mathbf{V}_{11} \in \mathbb{R}^{N\times N}$ and $\mathbf{V}_{22} \in \mathbb{R}^{NK\times NK}$ correspond to the asymptotic covariance matrix of $\sqrt{T}(\hat{\boldsymbol{\alpha}} - \boldsymbol{\alpha})$ and $\sqrt{T}(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta})$, respectively, whereas $\mathbf{V}_{12} \in \mathbb{R}^{N\times NK}$ is the asymptotic covariance matrix between $\sqrt{T}(\hat{\boldsymbol{\alpha}} - \boldsymbol{\alpha})$ and $\sqrt{T}(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta})$. Furthermore, we often refer to diagonal elements of $\mathbf{V}_{11}$ and $\mathbf{V}_{22}$ as asymptotic variances of the LSE of alphas and betas, respectively.

A straightforward calculation using (3) and (4) yields the following explicit forms of the block matrices:

$$\mathbf{V}_{11} = \sum_{l=-\infty}^{\infty} E[\{1 - \boldsymbol{\mu}'\mathbf{V}_f^{-1}(\mathbf{f}_t - \boldsymbol{\mu})\}\{1 - \boldsymbol{\mu}'\mathbf{V}_f^{-1}(\mathbf{f}_{t-l} - \boldsymbol{\mu})\}' \otimes \boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_{t-l}']$$

$$= \sum_{l=-\infty}^{\infty} E(\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_{t-l}') + (\boldsymbol{\mu}'\mathbf{V}_f^{-1} \otimes \mathbf{I}_N)$$

$$\times \sum_{l=-\infty}^{\infty} E\{(\mathbf{f}_t - \boldsymbol{\mu})(\mathbf{f}_{t-l} - \boldsymbol{\mu})' \otimes \boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_{t-l}'\}(\mathbf{V}_f^{-1}\boldsymbol{\mu} \otimes \mathbf{I}_N)$$

$$- (\boldsymbol{\mu}'\mathbf{V}_f^{-1} \otimes \mathbf{I}_N) \sum_{l=-\infty}^{\infty} E\{(\mathbf{f}_t - \boldsymbol{\mu}) \otimes \boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_{t-l}'\}$$

$$- \sum_{l=-\infty}^{\infty} E\{(\mathbf{f}_{t-l} - \boldsymbol{\mu})' \otimes \boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_{t-l}'\}(\mathbf{V}_f^{-1}\boldsymbol{\mu} \otimes \mathbf{I}_N)$$

$$= \sum_{l=-\infty}^{\infty} E(\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_{t-l}') + (\boldsymbol{\mu}' \otimes \mathbf{I}_N)\mathbf{V}_{22}(\boldsymbol{\mu} \otimes \mathbf{I}_N)$$

$$- (\boldsymbol{\mu}'\mathbf{V}_f^{-1} \otimes \mathbf{I}_N) \sum_{l=-\infty}^{\infty} E\{(\mathbf{f}_t - \boldsymbol{\mu}) \otimes \boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_{t-l}'\}$$

$$- \left[(\boldsymbol{\mu}'\mathbf{V}_f^{-1} \otimes \mathbf{I}_N) \sum_{l=-\infty}^{\infty} E\{(\mathbf{f}_t - \boldsymbol{\mu}) \otimes \boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_{t-l}'\}\right]',$$

$$\mathbf{V}_{12} = \sum_{l=-\infty}^{\infty} E[\{1 - \boldsymbol{\mu}'\mathbf{V}_f^{-1}(\mathbf{f}_t - \boldsymbol{\mu})\}(\mathbf{f}_{t-l} - \boldsymbol{\mu})'\mathbf{V}_f^{-1} \otimes \boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_{t-l}']$$

$$= \sum_{l=-\infty}^{\infty} E\{(\mathbf{f}_{t-l} - \boldsymbol{\mu})' \otimes \boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_{t-l}'\}(\mathbf{V}_f^{-1} \otimes \mathbf{I}_N)$$

$$- (\boldsymbol{\mu}'\mathbf{V}_f^{-1} \otimes \mathbf{I}_N)\sum_{l=-\infty}^{\infty} E\{(\mathbf{f}_t - \boldsymbol{\mu})(\mathbf{f}_{t-l} - \boldsymbol{\mu})' \otimes \boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_{t-l}'\}(\mathbf{V}_f^{-1} \otimes \mathbf{I}_N)$$

$$= \left[(\mathbf{V}_f^{-1} \otimes \mathbf{I}_N)\sum_{l=-\infty}^{\infty} E\{(\mathbf{f}_t - \boldsymbol{\mu}) \otimes \boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_{t-l}'\}\right]' - (\boldsymbol{\mu}' \otimes \mathbf{I}_N)\mathbf{V}_{22},$$

$$\mathbf{V}_{22} = \sum_{l=-\infty}^{\infty} E\{\mathbf{V}_f^{-1}(\mathbf{f}_t - \boldsymbol{\mu})(\mathbf{f}_{t-l} - \boldsymbol{\mu})'\mathbf{V}_f^{-1} \otimes \boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_{t-l}'\}$$

$$= (\mathbf{V}_f^{-1} \otimes \mathbf{I}_N)\sum_{l=-\infty}^{\infty} E\{(\mathbf{f}_t - \boldsymbol{\mu})(\mathbf{f}_{t-l} - \boldsymbol{\mu})' \otimes \boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_{t-l}'\}(\mathbf{V}_f^{-1} \otimes \mathbf{I}_N).$$

We can see that $\mathbf{V}_{22}$ depends only on $\sum_{l=-\infty}^{\infty} E\{(\mathbf{f}_t - \boldsymbol{\mu})(\mathbf{f}_{t-l} - \boldsymbol{\mu})' \otimes \boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_{t-l}'\}$, which is proportional to the long-run cokurtosis of $\mathbf{f}_t$ and $\boldsymbol{\varepsilon}_t$. In contrast, both $\mathbf{V}_{11}$ and $\mathbf{V}_{12}$ depend not only on $\sum_{l=-\infty}^{\infty} E\{(\mathbf{f}_t - \boldsymbol{\mu})(\mathbf{f}_{t-l} - \boldsymbol{\mu})' \otimes \boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_{t-l}'\}$ but also on $\sum_{l=-\infty}^{\infty} E\{(\mathbf{f}_t - \boldsymbol{\mu}) \otimes \boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_{t-l}'\}$, which is proportional to the long-run coskewness of $\mathbf{f}_t$ and $\boldsymbol{\varepsilon}_t$.

Therefore, while the asymptotic covariance matrix of the LSE of betas depends only on the long-run cokurtosis of factors and error terms, that of alphas depends not only on the long-run cokurtosis but also on the long-run coskewness of factors and error terms. It is worth mentioning that the asymptotic covariance matrix of the LSE of betas has nothing to do with any skewness measures. This implies that the result of Ando and Hodoshima [1] continues to hold when the underlying joint distribution of factors and error terms exhibits heteroskedasticity and autocorrelation of unknown form.

3. Reexamination of the Fama-French Three-factor Model

3.1. Data description

In this subsection, we illustrate how heteroskedasticity and autocorrelation of unknown form in $\mathbf{v}_t$ affect the asymptotic variance estimates of the LSE in the Fama-French three-factor model. The data set has been downloaded from Kenneth French's web page. Asset returns are 25 value-weighted returns on the intersections of 5 portfolios formed on size and 5 portfolios formed on the ratio of book equity to market equity. Factors include the excess return on the market $(R_m - R_f)$, the average return on the three small portfolios minus the average return on the three big portfolios (*SMB*), and the average return on the two value portfolios minus the average return on the two growth portfolios (*HML*). Hence, we can see that $(N, K) = (25, 3)$. Two data frequencies (monthly, daily) are considered, and sample periods are July 1963 - August 2008 and July 1, 1963 - August 29, 2008 for monthly and daily data, respectively. We remark that August 2008 is one month before the Lehman shock. We avoid including observations after the Lehman shock in order not to mix observations of different nature into the sample. As a consequence, numbers of observations are 542 and 11370 for monthly and daily data, respectively.

3.2. Estimation of the LRCM

To obtain estimates of the asymptotic variances of the LSE of alphas and betas, we must estimate the covariance matrix (2). The Hessian matrix $\mathbf{H}$ can be consistently estimated by its sample analog

$$\hat{\mathbf{H}} = \hat{\mathbf{F}} \otimes \mathbf{I}_N = \begin{bmatrix} 1 & (1/T)\sum_{t=1}^{T} \mathbf{f}_t' \\ (1/T)\sum_{t=1}^{T} \mathbf{f}_t & (1/T)\sum_{t=1}^{T} \mathbf{f}_t \mathbf{f}_t' \end{bmatrix} \otimes \mathbf{I}_N.$$

On the other hand, to estimate the LRCM $\mathbf{S}$, we employ HAC estimation. HAC estimation of S can be implemented as follows. First, because $\mathbf{v}_t$ is unobservable due to error terms $\boldsymbol{\varepsilon}_t$, it is replaced by $\hat{\mathbf{v}}_t \equiv \mathbf{X}_t \otimes \hat{\boldsymbol{\varepsilon}}_t$, where $\hat{\boldsymbol{\varepsilon}}_t \equiv \mathbf{R}_t - \hat{\boldsymbol{\Theta}}\mathbf{X}_t$ is the LSE residual. Second, $\boldsymbol{\Gamma}_v(l)$ (= the lth autocovariance of $\mathbf{v}_t$) can be estimated by its sample analog based on $\hat{\mathbf{v}}_t$, i.e.,

$$\hat{\boldsymbol{\Gamma}}_v(l) \equiv \frac{1}{T} \sum_{t=\max\{1,1+l\}}^{\min\{T+l,T\}} \hat{\mathbf{v}}_t \hat{\mathbf{v}}_{t-l}', \; l = 0, \pm 1, \ldots, \pm (T-1).$$

Third, given a kernel $k(\cdot)$ and a bandwidth $M(>0)$, we finally obtain the HAC estimator of S as

$$\hat{\mathbf{S}} = \sum_{l=-(T-1)}^{T-1} k\left(\frac{l}{M}\right) \hat{\boldsymbol{\Gamma}}_v(l).$$

Regularity conditions for the consistency of $\hat{\mathbf{S}}$ are given in Appendix. As a consequence, the asymptotic covariance matrix $\mathbf{V}$ can be consistently estimated by $\hat{\mathbf{V}} = \hat{\mathbf{H}}^{-1}\hat{\mathbf{S}}\hat{\mathbf{H}}^{-1}$.

Computing the HAC estimate $\hat{\mathbf{S}}$ requires us to choose the kernel $k(\cdot)$ and the bandwidth M. It is well-known that choosing the latter is more important than choosing the former. Hence, we first choose a kernel and then we adopt the bandwidth choice method that is expected to match most suitably with the kernel. Specifically, we consider the following three kernel and bandwidth combinations: (i) the Quadratic Spectral ("QS") kernel and Andrews' [2] bandwidth; (ii) the Bartlett ("BT") kernel and Newey and West's [10] bandwidth; and (iii) the Parzen ("PZ") kernel and Hirukawa's [6] bandwidth. Each of the first two methods relies on an explicit formula and is applied popularly in empirical works. On the other hand, the third method yields a bandwidth value via solving a nonlinear equation numerically. Monte Carlo simulations in Hirukawa [6] indicate superior performance of this method over other two methods in terms of LRCM estimation. A brief comparison of these three approaches can be also found in Section 2 of Hirukawa [7]. We finally make a few remarks on the details of implementing the methods.

(1) For Andrews' [2] bandwidth, a first-order autoregressive ("AR(1)") model is fitted to each element of $\hat{\mathbf{v}}_t$. The weight w_a in $\hat{\alpha}(2)$ (see equation (6.4) of Andrews [2]) takes zero for the first N elements of $\hat{\mathbf{v}}_t$ (that correspond to intercepts, i.e., alphas) and one for the rest.

(2) For Newey and West's [10] bandwidth, the $N(K+1)$-dimensional column vector of weights on $\hat{\mathbf{v}}_t$ (see p. 634 of Newey and West [10]) is set equal to $\mathbf{w} = (0, \ldots, 0, 1, \ldots, 1)'$, where the first N elements (that correspond to intercepts, i.e., alphas) are zeros. Also, the lag selection parameter (see equation (3.10) of Newey and West [10]) is set equal to $n = [4(T/100)^{2/9}]$, where $[\cdot]$ denotes the integer part.

(3) For Hirukawa's [6] bandwidth, the $N(K+1)$-dimensional weight vector $\mathbf{w}$ is the same as the one used for Newey and West's [10] bandwidth. Then, an AR(1) model is fitted to a scalar process $\mathbf{w}'\hat{\mathbf{v}}_t$ to obtain $\hat{\alpha}(2)$ on p. 718 of Hirukawa [6].

Table 1. Least-squares estimates and asymptotic variance estimates of Fama-French three-factor models (monthly data)

Panel (a):

	alphas						betas on $R_m - R_f$					
		Asymptotic variance estimate						Asymptotic variance estimate				
				HAC						HAC		
	LSE	Robust	Normal	QS	BT	PZ	LSE	Robust	Normal	QS	BT	PZ
A1	-0.008	5.227	5.470	5.359	8.214	6.777	1.071	0.388	0.342	0.387	0.502	0.446
A2	0.455	3.101	2.989	2.973	4.562	3.738	0.965	0.262	0.187	0.365	0.427	0.412
A3	0.464	1.970	1.991	1.992	3.247	2.456	0.921	0.163	0.124	0.160	0.211	0.179
A4	0.617	1.903	1.996	1.923	3.068	2.467	0.893	0.195	0.125	0.210	0.220	0.192
A5	0.586	2.077	2.123	2.110	2.338	2.316	0.976	0.193	0.133	0.229	0.240	0.250
B1	0.278	2.721	2.831	3.092	3.848	3.312	1.118	0.195	0.177	0.244	0.349	0.328
B2	0.382	2.286	2.364	2.436	3.783	2.968	1.029	0.181	0.148	0.214	0.274	0.262
B3	0.567	2.045	2.022	2.105	2.689	2.262	0.977	0.184	0.126	0.224	0.280	0.256
B4	0.545	2.021	1.924	2.352	3.613	2.764	0.976	0.127	0.120	0.131	0.140	0.141
B5	0.458	2.232	2.086	2.523	3.655	3.124	1.079	0.168	0.130	0.186	0.249	0.208
C1	0.414	2.529	2.551	2.548	2.623	2.356	1.082	0.212	0.159	0.207	0.230	0.214
C2	0.488	2.920	2.986	3.096	3.452	3.278	1.057	0.279	0.186	0.295	0.417	0.362
C3	0.445	2.611	2.797	2.907	3.376	3.193	1.018	0.268	0.175	0.348	0.512	0.461
C4	0.471	2.526	2.533	2.642	3.775	3.011	1.006	0.185	0.158	0.198	0.243	0.211
C5	0.509	3.172	3.225	3.222	3.441	3.464	1.098	0.306	0.201	0.370	0.401	0.441
D1	0.606	2.708	2.464	3.074	4.910	4.063	1.054	0.244	0.154	0.242	0.258	0.244
D2	0.332	3.079	3.163	3.679	4.467	4.189	1.096	0.350	0.197	0.401	0.448	0.452
D3	0.411	2.868	3.157	3.169	3.254	3.007	1.081	0.316	0.197	0.345	0.417	0.411
D4	0.512	2.685	2.757	2.496	2.681	2.476	1.036	0.278	0.172	0.247	0.288	0.279
D5	0.352	4.346	4.255	4.477	3.869	3.960	1.162	0.373	0.266	0.443	0.577	0.548
E1	0.665	1.615	1.604	1.822	2.423	2.186	0.955	0.151	0.100	0.185	0.252	0.228
E2	0.478	2.257	2.229	2.455	3.103	2.748	1.030	0.182	0.139	0.185	0.240	0.211
E3	0.382	3.257	3.087	3.794	3.983	3.745	0.989	0.259	0.193	0.312	0.396	0.362
E4	0.348	2.294	2.383	2.306	2.625	2.512	0.996	0.183	0.149	0.174	0.213	0.197
E5	0.273	4.613	4.888	4.911	5.403	4.989	1.059	0.479	0.305	0.535	0.840	0.718

Panel (b):

	betas on *SMB*						betas on *HML*					
		Asymptotic variance estimate						Asymptotic variance estimate				
				HAC						HAC		
	LSE	Robust	Normal	QS	BT	PZ	LSE	Robust	Normal	QS	BT	PZ
A1	1.364	1.089	0.574	0.963	1.043	0.893	-0.332	1.221	0.765	1.404	1.213	1.335
A2	1.307	0.964	0.313	1.394	1.824	1.732	0.051	0.948	0.418	1.094	1.459	1.428
A3	1.091	0.390	0.209	0.370	0.487	0.446	0.291	0.511	0.278	0.428	0.658	0.533
A4	1.028	0.469	0.209	0.491	0.470	0.429	0.453	0.488	0.279	0.452	0.622	0.507
A5	1.074	0.598	0.223	0.702	0.917	0.873	0.677	0.630	0.297	0.733	0.854	0.845
B1	0.980	0.521	0.297	0.598	0.656	0.673	-0.402	0.729	0.396	0.795	1.431	1.010
B2	0.861	0.556	0.248	0.805	1.418	1.280	0.162	0.791	0.331	1.195	2.908	2.097
B3	0.760	0.658	0.212	0.856	1.476	1.289	0.407	0.771	0.283	1.173	2.793	2.009
B4	0.712	0.283	0.202	0.342	0.611	0.524	0.579	0.500	0.269	0.697	1.777	1.210
B5	0.852	0.293	0.219	0.389	0.603	0.563	0.781	0.407	0.292	0.449	0.707	0.591
C1	0.715	0.481	0.268	0.453	0.438	0.416	-0.455	0.571	0.357	0.526	0.590	0.526
C2	0.513	0.910	0.313	1.382	2.394	2.277	0.213	1.089	0.418	1.648	4.124	2.862
C3	0.425	0.868	0.293	1.285	2.211	2.009	0.489	0.879	0.391	1.413	3.736	2.576
C4	0.379	0.556	0.266	0.800	1.522	1.330	0.662	0.793	0.354	1.275	3.328	2.287
C5	0.527	0.957	0.338	1.601	2.334	2.234	0.824	0.853	0.451	1.044	2.108	1.564
D1	0.363	0.898	0.258	0.857	0.648	0.738	-0.445	0.801	0.344	0.835	0.778	0.841
D2	0.200	0.901	0.332	1.288	1.890	1.796	0.247	1.118	0.442	1.784	4.329	3.085
D3	0.161	0.988	0.331	1.215	1.954	1.845	0.491	1.018	0.441	1.563	3.988	2.691
D4	0.211	0.545	0.289	0.457	0.348	0.391	0.609	0.736	0.386	1.008	1.801	1.466
D5	0.234	0.790	0.446	1.088	1.743	1.566	0.819	0.787	0.595	1.064	1.715	1.460
E1	-0.262	0.282	0.168	0.388	0.413	0.442	-0.389	0.411	0.224	0.504	0.842	0.699
E2	-0.234	0.382	0.234	0.441	0.701	0.617	0.130	0.721	0.312	0.932	2.237	1.517
E3	-0.237	0.559	0.324	0.622	0.793	0.760	0.303	0.857	0.432	0.996	1.501	1.274
E4	-0.220	0.314	0.250	0.375	0.606	0.538	0.613	0.573	0.333	0.739	1.784	1.284
E5	-0.095	1.113	0.513	1.096	0.983	1.018	0.785	1.405	0.683	1.242	0.834	0.844

Note: Each portfolio return is expressed as a combination of a letter denoting the size (A to E) and a number denoting the ratio of book equity to market equity (1 to 5), where A and 1 are the smallest and E and 5 are the largest. The LSE means the least-squares estimate of the parameters. "Robust" and "Normal" are obtained from diagonal elements of estimates of $\mathbf{V}_0$ and $\mathbf{V}_G$, respectively. "HAC" denotes diagonal elements of estimates of $\mathbf{V}$, where "QS", "BT" and "PZ" are HAC estimates using the Quadratic Spectral kernel and Andrews' [2] bandwidth, the Bartlett kernel and Newey and West's [10] bandwidth, and the Parzen kernel and Hirukawa's [6] bandwidth, respectively.

Table 2. Least-squares estimates and asymptotic variance estimates of Fama-French three-factor models (daily data)

Panel (a):

	alphas						betas on $R_m - R_f$					
		Asymptotic variance estimate						Asymptotic variance estimate				
				HAC						HAC		
	LSE	Robust	Normal	QS	BT	PZ	LSE	Robust	Normal	QS	BT	PZ
A1	-0.009	0.175	0.173	0.220	0.360	0.320	1.101	0.811	0.331	1.297	3.355	2.624
A2	0.018	0.112	0.110	0.120	0.163	0.158	0.984	0.484	0.211	0.734	1.752	1.382
A3	0.022	0.088	0.086	0.093	0.118	0.111	0.890	0.506	0.165	0.803	2.031	1.540
A4	0.030	0.073	0.071	0.079	0.114	0.105	0.846	0.465	0.137	0.949	2.483	1.810
A5	0.032	0.062	0.062	0.081	0.142	0.127	0.877	0.247	0.118	0.474	1.552	1.109
B1	0.007	0.119	0.117	0.130	0.163	0.151	1.176	0.406	0.225	0.583	1.518	1.082
B2	0.016	0.088	0.088	0.092	0.109	0.104	1.044	0.327	0.169	0.509	1.177	0.927
B3	0.026	0.077	0.077	0.081	0.095	0.094	0.984	0.272	0.148	0.490	1.433	1.048
B4	0.025	0.072	0.072	0.077	0.106	0.096	0.970	0.298	0.138	0.500	1.417	1.029
B5	0.020	0.095	0.094	0.093	0.120	0.105	1.122	0.663	0.181	1.122	2.593	2.164
C1	0.017	0.120	0.119	0.120	0.123	0.125	1.112	0.467	0.228	0.748	1.617	1.276
C2	0.024	0.089	0.089	0.112	0.137	0.137	0.995	0.266	0.171	0.500	1.150	0.901
C3	0.022	0.090	0.089	0.104	0.126	0.122	0.943	0.368	0.171	0.705	2.217	1.611
C4	0.024	0.092	0.093	0.107	0.128	0.127	0.949	0.518	0.178	1.183	3.586	2.738
C5	0.024	0.133	0.133	0.132	0.157	0.149	1.102	0.623	0.255	1.224	4.065	2.750
D1	0.026	0.120	0.120	0.116	0.141	0.125	1.079	0.501	0.230	0.716	1.729	1.368
D2	0.017	0.105	0.103	0.120	0.154	0.144	0.982	0.525	0.198	1.244	2.540	2.116
D3	0.020	0.102	0.102	0.124	0.134	0.139	0.979	0.560	0.195	1.289	3.239	2.750
D4	0.023	0.110	0.110	0.112	0.114	0.117	1.000	0.496	0.210	0.852	1.892	1.600
D5	0.017	0.181	0.178	0.189	0.182	0.189	1.106	0.819	0.341	1.143	3.100	2.270
E1	0.032	0.053	0.053	0.069	0.082	0.079	0.960	0.275	0.102	0.459	1.384	1.076
E2	0.023	0.096	0.094	0.098	0.111	0.106	0.983	0.491	0.179	0.600	1.141	0.981
E3	0.016	0.130	0.130	0.136	0.150	0.142	0.999	0.555	0.249	0.936	2.589	1.915
E4	0.014	0.120	0.117	0.113	0.114	0.113	1.018	0.682	0.223	0.712	1.160	0.986
E5	0.008	0.185	0.184	0.207	0.260	0.242	1.148	0.709	0.353	1.205	2.709	1.980

Panel (b):

	betas on *SMB*						betas on *HML*					
		Asymptotic variance estimate						Asymptotic variance estimate				
				HAC						HAC		
	LSE	Robust	Normal	QS	BT	PZ	LSE	Robust	Normal	QS	BT	PZ
A1	1.145	2.552	0.788	4.729	9.803	7.921	-0.001	3.497	1.224	5.501	15.542	10.498
A2	1.024	1.361	0.502	2.396	6.939	5.095	0.206	2.361	0.780	4.855	17.129	11.545
A3	0.884	0.788	0.393	1.131	2.900	2.123	0.343	1.807	0.611	3.588	15.703	10.045
A4	0.847	0.755	0.325	0.933	2.409	1.726	0.436	1.447	0.505	2.403	8.122	5.389
A5	0.862	0.768	0.281	1.196	2.761	2.085	0.564	1.019	0.437	2.131	8.004	5.306
B1	1.014	1.001	0.536	1.107	2.164	1.718	-0.208	2.244	0.832	4.027	14.730	9.543
B2	0.886	0.900	0.402	1.410	3.294	2.588	0.212	1.486	0.624	3.157	12.553	8.325
B3	0.840	0.695	0.353	1.235	4.235	2.940	0.378	1.321	0.549	2.548	9.357	6.272
B4	0.796	0.914	0.328	1.928	6.738	4.900	0.548	1.142	0.509	2.543	7.305	5.272
B5	0.873	0.992	0.431	1.756	4.974	3.835	0.773	2.132	0.669	4.440	13.984	10.783
C1	0.756	1.072	0.542	1.413	3.196	2.423	-0.380	1.856	0.842	2.678	4.575	3.839
C2	0.636	1.066	0.407	2.043	6.378	4.770	0.142	1.290	0.632	2.230	7.589	5.240
C3	0.570	1.505	0.407	3.162	10.128	7.302	0.400	1.487	0.632	3.289	11.906	8.033
C4	0.518	1.347	0.424	3.311	11.271	8.563	0.538	2.401	0.659	5.998	19.131	14.090
C5	0.553	1.926	0.607	2.823	7.211	5.603	0.761	2.065	0.942	3.724	12.218	8.232
D1	0.418	1.712	0.548	2.502	7.128	5.420	-0.367	2.307	0.851	4.404	11.802	8.963
D2	0.300	3.687	0.471	4.337	7.490	6.330	0.190	2.506	0.731	5.111	21.783	14.496
D3	0.285	3.336	0.465	4.035	8.128	6.742	0.422	2.877	0.722	7.271	26.153	19.368
D4	0.283	1.457	0.501	1.936	3.072	2.416	0.630	2.847	0.778	6.621	24.375	16.992
D5	0.258	3.475	0.811	4.616	7.291	6.290	0.800	2.763	1.261	4.421	15.235	10.034
E1	-0.346	0.666	0.243	1.184	3.582	2.765	-0.454	1.113	0.377	2.384	8.335	6.017
E2	-0.296	1.428	0.428	2.136	4.436	3.413	0.098	2.351	0.664	5.082	21.291	13.801
E3	-0.236	2.687	0.594	3.068	6.179	4.901	0.350	2.557	0.922	4.984	18.128	11.987
E4	-0.224	5.542	0.532	7.126	8.541	7.944	0.671	2.715	0.827	4.472	12.440	8.453
E5	-0.170	2.000	0.841	2.029	3.227	2.711	0.955	2.766	1.306	3.794	8.473	6.328

Note: Each portfolio return is expressed as a combination of a letter denoting the size (A to E) and a number denoting the ratio of book equity to market equity (1 to 5), where A and 1 are the smallest and E and 5 are the largest. The LSE means the least-squares estimate of the parameters. "Robust" and "Normal" are obtained from diagonal elements of estimates of V_0 and V_G, respectively. "HAC" denotes diagonal elements of estimates of V, where "QS", "BT" and "PZ" are HAC estimates using the Quadratic Spectral kernel and Andrews' [2] bandwidth, the Bartlett kernel and Newey and West's [10] bandwidth, and the Parzen kernel and Hirukawa's [6] bandwidth, respectively.

3.3. Estimation results

Tables 1-2 present the LSE of alphas and betas and their asymptotic variance estimates for monthly and daily data. Differences in the LSE between monthly and daily data are substantial in alphas but not much in betas. Typically, the LSE of alphas in monthly data are 20 times as large as that in daily data, which seems to reflect the difference in portfolio returns of the two data. Asymptotic variance estimates of $\mathbf{V}$ are computed based on three HAC estimates (i.e., QS, BT and PZ), as well as estimates of $\mathbf{V}_0$ (labeled as "Robust") and $\mathbf{V}_G$ (labeled as "Normal") that are valid in the absence of serial dependence in $\mathbf{v}_t$ and under the i.i.d. normal assumption on $(\varepsilon_t', \mathbf{f}_t')'$, respectively. Estimated bandwidth values for HAC estimation are 2.133 (QS), 12.131 (BT) and 8.956 (PZ) for monthly data, and 6.689 (QS), 65.800 (BT) and 49.628 (PZ) for daily data. After comparing three HAC estimates on a given LSE, we can see a general tendency in the order of QS, PZ and BT from the smallest to the largest in terms of the size of a variance estimate. It is also conspicuous that BT tends to generate *by far* the largest asymptotic variance in the estimation results of monthly and daily data.

Differences between the estimates of $\mathbf{V}$ and $\mathbf{V}_0$ (or $\mathbf{V}_G$) depend on data frequencies and factors. A quick examination reveals that as regards alphas, the differences are relatively small for each of monthly and daily data, whereas estimates of BT (in particular) and PZ (to a lesser extent) tend to take large values compared to those of $\mathbf{V}_G$, $\mathbf{V}_0$, and QS. As regards betas, in contrast, the differences in the size of variance estimates are more distinct and particularly remarkable in all three factors for daily data. For monthly data, the differences are small in the excess market return, whereas they are considerable in other two factors between the estimates of $\mathbf{V}$ (or $\mathbf{V}_0$) and $\mathbf{V}_G$. It appears that discrepancies in the estimates of $\mathbf{V}$ and $\mathbf{V}_0$ rather depend on the choice of portfolio and combination of kernel and bandwidth. For daily data, the size differences of variance estimates are large for all three factors, in particular between $\mathbf{V}$ (or $\mathbf{V}_0$) and $\mathbf{V}_G$. There are also substantial differences between $\mathbf{V}$ and $\mathbf{V}_0$. These findings suggest that we should take effects of nonnormality and serial dependence into account, in particular, when evaluating precision of the LSE of betas from daily data.

4. Conclusion

We have derived the asymptotic covariance matrix formulas for the LSE of alphas and betas under heteroskedasticity and autocorrelation of unknown form in a generic multifactor asset pricing model. Particular attention has been paid to how nonnormality and autocorrelation affect the asymptotic covariance matrix of the LSE of alphas and betas. It is demonstrated that the asymptotic covariance matrix of the LSE of betas depends only on the long-run cokurtosis of factors and error terms, whereas that of alphas depends not only on the long-run cokurtosis but also on the long-run coskewness of factors and error terms. It is worth noting that the asymptotic covariance matrix for betas is free of skewness measures. We have also reexamined the robustness of the benchmark Fama-French three-factor model using the U.S. monthly and daily data when HAC estimators are employed for the LRCM estimation.

Examining empirical models under alternative scenarios of the underlying DGP, namely, i.i.d. normal, i.i.d. nonnormal, and heteroskedasticity and autocorrelation of unknown form, has been established in econometrics for many years. The exercise is also quite useful in asset pricing modeling to extract information, as we did in this note, about whether particular assumptions on the underlying DGP are satisfied or not, or more specifically about how nonnormality and serial dependence affects the asymptotic variance estimates of alphas and betas. Quite nicely, it is not laborious! We hope that our reexamination of the Fama-French three-factor model serves as a good empirical exercise of the robustness study.

A. Appendix: Regularity Conditions on HAC Estimation

For the HAC estimator $\hat{\mathbf{S}}$ (and thus $\hat{\mathbf{V}}$) to be consistent, we need the following assumptions. A sufficient condition for Assumption 1 is strong mixing with some size plus moment bounds. Popular choices of kernels such as QS, BT and PZ satisfy Assumption 2. In particular, $q = 1$ for BT and $q = 2$ for QS, PZ. Assumption 3 requires a (deterministic) sequence of the bandwidth M to diverge to infinity at a slower rate than the sample size T.

In reality, however, most applications including this paper consider an automatic (i.e., a data-driven) bandwidth, which is a stochastic sequence by construction. For such a stochastic bandwidth to deliver consistency in HAC estimation, we must impose additional technical conditions, which vary across three bandwidth choice methods considered in this paper. To save space, we concentrate on regularity conditions when the bandwidth M is a deterministic sequence.

Assumption 1. The process $\mathbf{v}_t$ is a zero-mean, fourth-order stationary sequence that satisfies

$$\sum_{l=-\infty}^{\infty} | l |^q \| \Gamma_v(l) \| < \infty$$

and

$$\sum_{a=-\infty}^{\infty} \sum_{b=-\infty}^{\infty} \sum_{c=-\infty}^{\infty} | \kappa_{i_1,i_2,i_3,i_4}(a, b, c) | < \infty,$$

where $\| \mathbf{A} \|$ signifies the Euclidean norm of matrix $\mathbf{A}$, i.e., $\| \mathbf{A} \| = \{tr(\mathbf{A}'\mathbf{A})\}^{1/2}$, $q \in (0, \infty)$ is the characteristic exponent of the kernel $k(\cdot)$ (Parzen [11]) that satisfies

$$k_q \equiv \lim_{x \to 0} \frac{1 - k(x)}{| x |^q} \in (0, \infty),$$

$\Gamma_v(l)$ denotes the lth autocovariance of $\mathbf{v}_t$, $\kappa_{i_1,i_2,i_3,i_4}(a, b, c)$ is the fourth-order cumulant of $(\mathbf{v}_{i_1,t}, \mathbf{v}_{i_2,t+a}, \mathbf{v}_{i_3,t+a+b}, \mathbf{v}_{i_4,t+a+b+c})$, and $\mathbf{v}_{i,t}$ is the ith element of $\mathbf{v}_t$.

Assumption 2. The kernel $k(\cdot)$ satisfies $k : \mathbb{R} \to [-1, 1]$, $k(0) = 1$, $k(x) = k(-x)$, $\forall x \in \mathbb{R}$, $k(\cdot)$ is continuous at 0 and almost everywhere, and $\int_0^\infty \bar{k}(x)dx < \infty$, where $\bar{k}(x) = \sup_{y \geq x} | k(y) |$.

Assumption 3. The bandwidth $M(= M_T)$ satisfies $1/M + M^q/T \to 0$ as $T \to \infty$.

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