

# Supplement to “Consistent Estimation of Linear Regression Models Using Matched Data”

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## A    Remainder of Proof of Theorem 3: Derivation of Asymptotic Variances

This section derives asymptotic variances  $V_I$ ,  $V_{II}$  and  $V_{III}$  in Theorem 3. Denote

$$\begin{aligned}\Omega_{11} &= \lim_{\substack{n, m \rightarrow \infty \\ n/m \rightarrow \kappa \in [0, \infty)}} \text{Var}(\sqrt{n}E_{RW}), \\ \Omega_{12} &= \lim_{\substack{n, m \rightarrow \infty \\ n/m \rightarrow \kappa \in (0, \infty)}} \text{Cov}(\sqrt{n}E_{RW}, \sqrt{m}E_{\Sigma_2}), \text{ and} \\ \Omega_{22} &= \lim_{m \rightarrow \infty} \text{Var}(\sqrt{m}E_{\Sigma_2}).\end{aligned}$$

Then,  $V_I = P_W^{-1}\Omega P_W^{-1}$ ,  $V_{II} = P_W^{-1}\Omega_{11}P_W^{-1}$  and  $V_{III} = P_W^{-1}\Omega_{22}P_W^{-1}$ , where

$$\Omega := \Omega_{11} + \sqrt{\kappa}(\Omega_{12} + \Omega'_{12}) + \kappa\Omega_{22}.$$

In what follows, we derive  $\Omega_{11}$ ,  $\Omega_{12}$ , and  $\Omega_{22}$ .

(i)  $\Omega_{22}$ : Assume without loss of generality that  $\mathcal{S}_2$  is an ordered sample, i.e.,  $\mathcal{S}_2 = \{X_{2j}, Z_j\}_{j=1}^m = \{X_{2(j)}, Z_{(j)}\}_{j=1}^m$ . Lemma A3 implies that

$$\sqrt{m}E_{\Sigma_2} = \sum_{j=2}^m \frac{1}{\sqrt{m}} \left\{ \frac{1}{K} \left( \frac{\Delta\eta_{2j}\Delta\eta'_{2j}}{2} - \Sigma_2 \right) \beta_2 \right\} + o_p(m^{-1/2}).$$

Because  $\{(\Delta\eta_{2j}\Delta\eta'_{2j}/2) - \Sigma_2\} \beta_2$  is one-dependent, it is easy to see that  $\Omega_{22} = (1/K^2) \text{diag} \{0_{(d_1+1)\times(d_1+1)}, \Gamma(-1) + \Gamma(0) + \Gamma(1), 0_{d_3\times d_3}\}$ , where

$$\Gamma(\ell) = E \left\{ \left( \frac{\Delta\eta_{2j}\Delta\eta'_{2j}}{2} - \Sigma_2 \right) \beta_2 \beta_2' \left( \frac{\Delta\eta_{2j-\ell}\Delta\eta'_{2j-\ell}}{2} - \Sigma_2 \right) \right\}$$

is the  $\ell$ th autocovariance of  $\{(\Delta\eta_{2j}\Delta\eta'_{2j}/2) - \Sigma_2\} \beta_2$ . Furthermore, a straightforward calculation yields

$$\Gamma(0) = \frac{1}{2} E \{(\eta_2\eta_2' - \Sigma_2) \beta_2 \beta_2' (\eta_2\eta_2' - \Sigma_2)\} + \frac{1}{2} \{(\beta_2 \Sigma_2 \beta_2') \Sigma_2 + \Sigma_2 \beta_2 \beta_2' \Sigma_2\} := \frac{1}{2} \Xi + \frac{1}{4} \Psi$$

and  $\Gamma(\pm 1) = (1/4) \Xi$ . Therefore,

$$\Omega_{22} = \frac{1}{K^2} \text{diag} \left\{ 0_{(d_1+1)\times(d_1+1)}, \Xi + \frac{1}{2} \Psi, 0_{d_3\times d_3} \right\}.$$

(ii)  $\Omega_{11}$ : Define  $\phi_{i,j(i)} := W_{i,j(i)} (u_i + \eta'_{2i} \beta_2)$  and

$$\psi_{i,j(i)} := W_{i,j(i)} \eta'_{2j(i)} \beta_2 - \frac{1}{K} \Sigma \theta = \begin{bmatrix} \eta'_{2j(i)} \beta_2 \\ X_{1i} \eta'_{2j(i)} \beta_2 \\ \left( X_{2j(i)} \eta'_{2j(i)} - \frac{1}{K} \Sigma_2 \right) \beta_2 \\ Z_i \eta'_{2j(i)} \beta_2 \end{bmatrix} := \begin{bmatrix} \psi_{i,j(i),0} \\ \psi_{i,j(i),1} \\ \psi_{i,j(i),2} \\ \psi_{i,j(i),3} \end{bmatrix}.$$

Then,

$$\sqrt{n} E_{RW} = \sum_{i=1}^n \frac{1}{\sqrt{n}} \phi_{i,j(i)} - \sum_{i=1}^n \frac{1}{\sqrt{n}} \psi_{i,j(i)}.$$

It is easy to check that  $E(\phi_{i,j(i)} \phi'_{h,j(h)}) = E(\phi_{i,j(i)} \psi'_{h,j(h)}) = 0_{(d+1)\times(d+1)}$  for  $i \neq h$ .

Hence,  $\Omega_{11} = \Omega_{11A} + \Omega_{11B}$ , where

$$\Omega_{11A} := \text{Var}(\phi_{i,j(i)} - \psi_{i,j(i)}) = E \left\{ \left( W_{i,j(i)} \epsilon_{i,j(i)} + \frac{1}{K} \Sigma \theta \right) \left( W_{i,j(i)} \epsilon_{i,j(i)} + \frac{1}{K} \Sigma \theta \right)' \right\}$$

and  $\Omega_{11B}$  is the long-run variance of  $\psi_{i,j(i)}$  minus  $\text{Var}(\psi_{i,j(i)})$ .

To derive  $\Omega_{11B}$ , suppose that  $\psi_{i,j(i)}$  and  $\psi_{h,j(h)}$  ( $i \neq h$ ) have the unit  $j$  in  $\mathcal{S}_2$  in common. Because the probability that they have no other units in  $\mathcal{S}_2$  in common, conditional on sharing the unit  $j$ , is  $1 - (K-1)/(m-1) = 1 - O(m^{-1})$ ,

we may safely concentrate on the case in which the unit  $j$  is the only source of generating the covariance between them. Then, we find the terms involving the unit  $j$  that have non-zero expectations in  $\psi_{i,j(i)}\psi'_{h,j(h)}$ . Obviously,  $\eta'_{2j}\beta_2\beta'_2\eta_{2j}/K^2$  in  $\psi_{i,j(i),0}\psi'_{h,j(h),0}$ ,  $X_{1i}\eta'_{2j}\beta_2\beta'_2\eta_{2j}X_{1h}/K^2$  in  $\psi_{i,j(i),1}\psi'_{h,j(h),1}$ ,  $X_{1i}\eta'_{2j}\beta_2\beta'_2\eta_{2j}Z_h/K^2$  in  $\psi_{i,j(i),1}\psi'_{h,j(h),3}$ , and  $Z_i\eta'_{2j}\beta_2\beta'_2\eta_{2j}Z_h/K^2$  in  $\psi_{i,j(i),3}\psi'_{h,j(h),3}$  have non-zero expectations, which are  $\beta_2\Sigma_2\beta'_2/K^2$ ,  $(\beta_2\Sigma_2\beta'_2) E(X_1) E(X_1)' / K^2$ ,  $(\beta_2\Sigma_2\beta'_2) E(X_1) E(Z)' / K^2$ , and  $(\beta_2\Sigma_2\beta'_2) E(Z) E(Z)' / K^2$ , respectively. For  $\psi_{i,j(i),0}\psi'_{h,j(h),2}$ ,  $\psi_{i,j(i),1}\psi'_{h,j(h),2}$  and  $\psi_{i,j(i),3}\psi'_{h,j(h),2}$ , write  $g_{2j(i)} = (1/K) \sum_{j \in \mathcal{J}_K(i)} g_2(Z_j)$ . The terms with non-zero expectations are  $\eta'_{2j}\beta_2\beta'_2\eta_{2j}g'_{2j(h)}/K^2$ ,  $X_{1i}\eta'_{2j}\beta_2\beta'_2\eta_{2j}g'_{2j(h)}/K^2$  and  $Z_i\eta'_{2j}\beta_2\beta'_2\eta_{2j}g'_{2j(h)}/K^2$ , and their expectations are  $(\beta_2\Sigma_2\beta'_2) E(X_2)' / K^2$ ,  $(\beta_2\Sigma_2\beta'_2) E(X_1) E(X_2)' / K^2$  and  $(\beta_2\Sigma_2\beta'_2) E(Z) E(X_2)' / K^2$ , respectively, due to  $X_{2j(h)} = g_{2j(h)} + \eta_{2j(h)}$ . Finally, recognizing that the terms including the unit  $j$  in  $\psi_{i,j(i),2}$  are

$$\frac{1}{K^2} \left\{ g_2(Z_j) \eta'_{2j}\beta_2 + \sum_{\ell \in \mathcal{J}_K(i), \ell \neq j} g_2(Z_\ell) \eta'_{2j}\beta_2 + (\eta_{2j}\eta'_{2j} - \Sigma_2) \beta_2 + \eta_{2j} \sum_{\ell \in \mathcal{J}_K(i), \ell \neq j} \eta'_{2\ell} \right\},$$

we obtain the terms with non-zero expectations in  $\psi_{i,j(i),2}\psi'_{h,j(h),2}$  as

$$\begin{aligned} & \frac{1}{K^4} \left\{ g_2(Z_j) \eta'_{2j}\beta_2\beta'_2\eta_{2j}g_2(Z_j) + \sum_{\ell \in \mathcal{J}_K(i), \ell \neq j} g_2(Z_\ell) \eta'_{2j}\beta_2\beta'_2\eta_{2j}g_2(Z_j) \right. \\ & + g_2(Z_j) \eta'_{2j}\beta_2\beta'_2\eta_{2j} \sum_{\ell \in \mathcal{J}_K(h), \ell \neq j} g_2(Z_\ell) + \sum_{\ell \in \mathcal{J}_K(i), \ell \neq j} g_2(Z_\ell) \eta'_{2j}\beta_2\beta'_2\eta_{2j} \sum_{\ell \in \mathcal{J}_K(h), \ell \neq j} g_2(Z_\ell) \\ & \left. + (\eta_{2j}\eta'_{2j} - \Sigma_2) \beta_2\beta'_2 (\eta_{2j}\eta'_{2j} - \Sigma_2) \right\}, \end{aligned}$$

which has the expected value

$$\frac{1}{K^2} \left[ (\beta_2\Sigma_2\beta'_2) E(X_2) E(X_2)' + \frac{1}{K^2} \{ (\beta_2\Sigma_2\beta'_2) \text{Var}(g_2(Z)) + \Xi \} \right].$$

Let  $N_K(j)$  be the number of times the unit  $j$  in  $\mathcal{S}_2$  is chosen as a match, i.e.,  $N_K(j) := \sum_{i=1}^n \mathbf{1}\{j \in \mathcal{J}_K(i)\}$ . Then, the unit  $j$  appears  $N_K(j) \{N_K(j) - 1\}$  times

among all covariance calculations as above. Since  $N_K(j) \sim \text{Bin}(n, K/m)$ ,

$$E[N_K(j) \{N_K(j) - 1\}] = K^2 \binom{n}{m} \left( \frac{n}{m} - \frac{1}{m} \right).$$

In conclusion,

$$\begin{aligned} \Omega_{11B} &= \lim_{\substack{n, m \rightarrow \infty \\ n/m \rightarrow \kappa \in [0, \infty)}} \sum_{j=1}^m K^2 \binom{n}{m} \left( \frac{n}{m} - \frac{1}{m} \right) \left( \frac{1}{\sqrt{n}} \right)^2 \\ &\quad \times \frac{1}{K^2} \begin{bmatrix} 1 \\ (\beta_2 \Sigma_2 \beta_2') \begin{bmatrix} E(X_1) \\ E(X_2) \\ E(Z) \end{bmatrix} \end{bmatrix} \begin{bmatrix} 1 & E(X_1)' & E(X_2)' & E(Z)' \end{bmatrix} \\ &\quad + \frac{1}{K^2} \text{diag} \left\{ 0_{(d_1+1) \times (d_1+1)}, (\beta_2 \Sigma_2 \beta_2') \text{Var} \{g_2(Z)\} + \Xi, 0_{d_3 \times d_3} \right\} \left\{ 1 + O(m^{-1}) \right\} \\ &= \begin{cases} \kappa \left[ (\beta_2 \Sigma_2 \beta_2') E(W) E(W)' \right. \\ \quad \left. + \frac{1}{K^2} \text{diag} \left\{ 0_{(d_1+1) \times (d_1+1)}, (\beta_2 \Sigma_2 \beta_2') \text{Var} \{g_2(Z)\} + \Xi, 0_{d_3 \times d_3} \right\} \right] & \text{if } n/m \rightarrow \kappa \in (0, \infty) \quad , \\ 0_{(d+1) \times (d+1)} & \text{if } n/m \rightarrow 0 \end{cases} \end{aligned}$$

which implies that  $\Omega_{11} = \Omega_{11A}$  if  $n/m \rightarrow 0$ .

(iii)  $\Omega_{12}$ : Obviously,  $E[\phi_{i,j(i)} \beta_2' \{(\Delta \eta_{2\ell} \Delta \eta_{2\ell}' / 2) - \Sigma_2\}] = 0_{(d+1) \times d_2}$  for any  $i, \ell$ . On the other hand, when  $\psi_{i,j(i)}$  includes the unit  $j$ ,  $\psi_{i,j(i)} \beta_2' [(1/K) \{(\Delta \eta_{2j} \Delta \eta_{2j}' / 2) - \Sigma_2\}]$  and  $\psi_{i,j(i)} \beta_2' [(1/K) \{(\Delta \eta_{2j+1} \Delta \eta_{2j+1}' / 2) - \Sigma_2\}]$  have terms with non-zero expectations. For each of these, the only correlated term is  $(2K^3)^{-1} (\eta_{2j} \eta_{2j}' - \Sigma_2) \beta_2 \beta_2' (\eta_{2j} \eta_{2j}' - \Sigma_2)$ .

Because the unit  $j$  appears  $N_K(j)$  times among all such covariance calculations and

$E\{N_K(j)\} = K(n/m)$ , (the negative of) the (2, 2) block of  $\Omega_{12}$  is given by

$$\lim_{\substack{n, m \rightarrow \infty \\ n/m \rightarrow \kappa \in (0, \infty)}} \sum_{j=1}^m K \binom{n}{m} \frac{1}{\sqrt{mn}} 2 \cdot \frac{1}{2K^3} \Xi \left\{ 1 + O(m^{-1}) \right\} = \frac{\sqrt{\kappa}}{K^2} \Xi,$$

which completes the proof. ■

**Remark.** The fact that  $\Omega_{11B} = 0_{(d+1) \times (d+1)}$  when  $n/m \rightarrow 0$  can be interpreted as follows. If  $m \gg n$ , then there are quite a few candidates of matches in  $\mathcal{S}_2$  for the unit  $i$  in  $\mathcal{S}_1$ . Sets of  $K$  matches chosen for units  $i$  and  $h (\neq i)$  become different, and

as a consequence  $N_K(j)$  becomes at most one. In this environment,  $\psi_{i,j(i)}$  and  $\psi_{h,j(h)}$  tend to have no units from  $\mathcal{S}_2$  in common, and  $\Omega_{11B} = 0_{(d+1) \times (d+1)}$  follows.

## B Comprehensive Simulation Results

This section provides a comprehensive version of simulation results in Section 4 including Models A-C. Tables B1-B3 present the results for  $d_3 = 1, 2, 3$ , respectively. Letters “A” and “B” right after MSOLS, MSII and MSII-FM indicate that  $W_{ij(i)}$  and  $W_{ij(i)}^\dagger$  are chosen as the regressor, respectively. For each estimator, the following performance measures are computed: (i) *Mean* (simulation average of the parameter estimate); (ii) *SD* (simulation standard deviation of the parameter estimate); (iii) *RMSE* (root mean-squared error of the parameter estimate); (iv)  $\overline{SE}$  (simulation average of the standard error); and (v) *CR* (coverage rate for the nominal 95% confidence interval). Since MSOLS is inconsistent and limiting distributions of the initial MSII for  $d_3 = 2, 3$  (denoted as “*initial*”) are not available, their standard errors are not well defined. Accordingly,  $\overline{SE}$  and *CR* are not computed for these estimators. Furthermore, only results of MSII-FM for  $K = 1$  are presented, because those for  $K \geq 2$  are quite poor.

**Table B1:** Monte Carlo Results ( $d_3 = 1$ )

**Model A:**  $g_{22}(z) = z + (5/\tau)\phi(z/\tau)$ ,  $\tau = 0.75$

$(n, m)$	Estimator	$\beta_{22}$					$\gamma_1$					
(1000, 500) $\Rightarrow \kappa = 2$	OLS*	Mean	1.0000					1.0003				
		SD	0.0266					0.0592				
		RMSE	0.0266					0.0592				
		$\overline{SE}$	0.0259					0.0571				
		CR	95%					94%				
		K	1	2	4	8	16	1	2	4	8	16
MSOLS-A	Mean	0.4362	0.5736	0.7167	0.8257	0.8781	1.5193	1.3820	1.2513	1.1527	1.1029	
	SD	0.0747	0.0831	0.0967	0.1096	0.1192	0.1409	0.1352	0.1379	0.1441	0.1482	
	RMSE	0.5687	0.4345	0.2993	0.2058	0.1705	0.5380	0.4052	0.2866	0.2100	0.1804	
MSOLS-B	Mean	0.4362	0.5736	0.7168	0.8259	0.8786	1.5190	1.3817	1.2510	1.1526	1.1033	
	SD	0.0748	0.0831	0.0967	0.1096	0.1194	0.1409	0.1353	0.1380	0.1443	0.1487	
	RMSE	0.5687	0.4344	0.2992	0.2057	0.1703	0.5378	0.4049	0.2865	0.2100	0.1811	
MSII-A	K	1	2	4	8	16	1	2	4	8	16	
	Mean	1.0766	1.0485	1.0337	1.0157	0.9816	0.9364	0.9625	0.9752	0.9886	1.0140	
	SD	0.3625	0.2489	0.1872	0.1594	0.1451	0.3385	0.2414	0.1958	0.1756	0.1643	
	RMSE	0.3705	0.2536	0.1902	0.1602	0.1462	0.3444	0.2443	0.1974	0.1759	0.1649	
	$\overline{SE}$	0.3383	0.1925	0.1368	0.1168	0.1091	0.3112	0.1941	0.1518	0.1368	0.1308	
MSII-B	Mean	1.0767	1.0487	1.0339	1.0159	0.9821	0.9360	0.9621	0.9748	0.9883	1.0142	
	SD	0.3624	0.2489	0.1872	0.1595	0.1452	0.3385	0.2414	0.1959	0.1757	0.1649	
	RMSE	0.3705	0.2536	0.1903	0.1603	0.1463	0.3445	0.2443	0.1975	0.1761	0.1655	
	$\overline{SE}$	0.3383	0.1925	0.1368	0.1168	0.1092	0.3113	0.1941	0.1518	0.1368	0.1311	
	CR	96%	91%	86%	86%	85%	96%	91%	87%	87%	87%	
(1000, 1000) $\Rightarrow \kappa = 1$	OLS*	Mean	1.0017					0.9955				
		SD	0.0256					0.0575				
		RMSE	0.0257					0.0577				
		$\overline{SE}$	0.0259					0.0570				
		CR	95%					95%				
		K	1	2	4	8	16	1	2	4	8	16
MSOLS-A	Mean	0.4376	0.5752	0.7151	0.8287	0.9005	1.5147	1.3768	1.2488	1.1501	1.0858	
	SD	0.0636	0.0746	0.0819	0.0912	0.0977	0.1266	0.1219	0.1230	0.1257	0.1268	
	RMSE	0.5660	0.4313	0.2964	0.1941	0.1395	0.5301	0.3961	0.2775	0.1958	0.1531	
MSOLS-B	Mean	0.4376	0.5752	0.7151	0.8287	0.9006	1.5147	1.3768	1.2487	1.1500	1.0860	
	SD	0.0636	0.0746	0.0819	0.0912	0.0978	0.1266	0.1220	0.1231	0.1258	0.1270	
	RMSE	0.5660	0.4313	0.2964	0.1941	0.1394	0.5300	0.3960	0.2775	0.1958	0.1534	
MSII-A	K	1	2	4	8	16	1	2	4	8	16	
	Mean	1.0427	1.0280	1.0186	1.0123	1.0025	0.9621	0.9753	0.9841	0.9913	0.9980	
	SD	0.2763	0.1932	0.1473	0.1257	0.1157	0.2591	0.1923	0.1608	0.1454	0.1369	
	RMSE	0.2796	0.1952	0.1485	0.1264	0.1158	0.2618	0.1938	0.1616	0.1456	0.1369	
	$\overline{SE}$	0.2673	0.1664	0.1269	0.1120	0.1058	0.2553	0.1757	0.1454	0.1338	0.1289	
MSII-B	Mean	1.0428	1.0280	1.0186	1.0124	1.0026	0.9621	0.9753	0.9840	0.9913	0.9981	
	SD	0.2763	0.1932	0.1473	0.1258	0.1158	0.2591	0.1922	0.1608	0.1454	0.1371	
	RMSE	0.2796	0.1952	0.1485	0.1264	0.1158	0.2619	0.1938	0.1616	0.1457	0.1371	
	$\overline{SE}$	0.2673	0.1664	0.1269	0.1120	0.1058	0.2553	0.1757	0.1454	0.1339	0.1290	
	CR	97%	92%	91%	93%	93%	96%	93%	92%	92%	94%	

**Table B1: Continued**

**Model A: Continued**

$(n, m)$	Estimator	$\beta_{22}$					$\gamma_1$					
(1000, 2000) $\Rightarrow \kappa = 1/2$	OLS*	Mean	1.0005					0.9974				
		SD	0.0261					0.0580				
		RMSE	0.0261					0.0581				
		$\overline{SE}$	0.0260					0.0571				
		CR	95%					94%				
		<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>
	MSOLS-A	Mean	0.4372	0.5733	0.7144	0.8313	0.9084	1.5131	1.3787	1.2499	1.1472	1.0793
		SD	0.0607	0.0682	0.0779	0.0827	0.0866	0.1273	0.1218	0.1208	0.1182	0.1207
		RMSE	0.5661	0.4321	0.2960	0.1879	0.1260	0.5287	0.3978	0.2776	0.1888	0.1444
	MSOLS-B	Mean	0.4372	0.5733	0.7144	0.8313	0.9084	1.5131	1.3787	1.2499	1.1472	1.0794
		SD	0.0607	0.0682	0.0779	0.0827	0.0865	0.1273	0.1218	0.1208	0.1182	0.1206
		RMSE	0.5661	0.4321	0.2960	0.1879	0.1260	0.5287	0.3978	0.2776	0.1888	0.1444
	<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	
MSII-A	Mean	1.0421	1.0235	1.0172	1.0152	1.0112	0.9623	0.9807	0.9861	0.9886	0.9911	
	SD	0.2415	0.1679	0.1324	0.1111	0.1013	0.2419	0.1814	0.1529	0.1342	0.1289	
	RMSE	0.2451	0.1695	0.1335	0.1121	0.1020	0.2448	0.1824	0.1535	0.1346	0.1292	
	$\overline{SE}$	0.2369	0.1557	0.1234	0.1106	0.1052	0.2317	0.1678	0.1429	0.1328	0.1284	
	CR	95%	93%	92%	96%	96%	95%	94%	94%	95%	96%	
MSII-B	Mean	1.0421	1.0235	1.0173	1.0152	1.0112	0.9623	0.9806	0.9861	0.9886	0.9911	
	SD	0.2415	0.1679	0.1324	0.1111	0.1013	0.2419	0.1814	0.1529	0.1342	0.1289	
	RMSE	0.2451	0.1695	0.1335	0.1121	0.1019	0.2448	0.1824	0.1535	0.1347	0.1292	
	$\overline{SE}$	0.2369	0.1557	0.1234	0.1106	0.1052	0.2317	0.1678	0.1429	0.1328	0.1284	
	CR	95%	93%	92%	96%	96%	95%	94%	94%	95%	96%	
(2000, 1000) $\Rightarrow \kappa = 2$	OLS*	Mean	0.9995					0.9983				
		SD	0.0179					0.0407				
		RMSE	0.0179					0.0408				
		$\overline{SE}$	0.0183					0.0404				
		CR	95%					96%				
		<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>
	MSOLS-A	Mean	0.4352	0.5703	0.7090	0.8244	0.8980	1.5143	1.3794	1.2525	1.1498	1.0850
		SD	0.0542	0.0600	0.0665	0.0723	0.0781	0.0998	0.0958	0.0979	0.0992	0.1020
		RMSE	0.5674	0.4339	0.2985	0.1899	0.1285	0.5239	0.3913	0.2709	0.1797	0.1328
	MSOLS-B	Mean	0.4352	0.5703	0.7090	0.8244	0.8981	1.5142	1.3793	1.2525	1.1498	1.0851
		SD	0.0542	0.0600	0.0665	0.0723	0.0781	0.0998	0.0958	0.0979	0.0992	0.1020
		RMSE	0.5674	0.4339	0.2985	0.1899	0.1284	0.5238	0.3912	0.2708	0.1797	0.1329
	<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	
MSII-A	Mean	1.0270	1.0147	1.0079	1.0066	0.9997	0.9750	0.9868	0.9929	0.9927	0.9977	
	SD	0.2171	0.1581	0.1220	0.1029	0.0944	0.2044	0.1561	0.1313	0.1174	0.1119	
	RMSE	0.2188	0.1588	0.1222	0.1031	0.0944	0.2060	0.1567	0.1315	0.1176	0.1119	
	$\overline{SE}$	0.2120	0.1273	0.0927	0.0798	0.0749	0.1971	0.1302	0.1043	0.0948	0.0911	
	CR	95%	88%	86%	87%	87%	96%	92%	88%	89%	90%	
MSII-B	Mean	1.0271	1.0148	1.0080	1.0066	0.9998	0.9749	0.9867	0.9928	0.9927	0.9978	
	SD	0.2171	0.1582	0.1220	0.1029	0.0945	0.2044	0.1562	0.1313	0.1174	0.1119	
	RMSE	0.2188	0.1589	0.1222	0.1031	0.0945	0.2060	0.1567	0.1315	0.1176	0.1119	
	$\overline{SE}$	0.2120	0.1273	0.0927	0.0798	0.0749	0.1971	0.1302	0.1043	0.0948	0.0911	
	CR	95%	88%	86%	87%	87%	96%	92%	88%	89%	90%	

**Table B1: Continued**

**Model A: Continued**

$(n, m)$	Estimator	$\beta_{22}$						$\gamma_1$					
(2000, 2000) $\implies \kappa = 1$	OLS*	<i>Mean</i>	1.0005						0.9984				
		<i>SD</i>	0.0178						0.0395				
		<i>RMSE</i>	0.0178						0.0396				
		$\overline{SE}$	0.0183						0.0404				
		<i>CR</i>	96%						95%				
		<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	
	MSOLS-A	<i>Mean</i>	0.4367	0.5726	0.7136	0.8268	0.9044	1.5130	1.3767	1.2484	1.1483	1.0806	
		<i>SD</i>	0.0470	0.0527	0.0580	0.0642	0.0679	0.0907	0.0878	0.0872	0.0887	0.0906	
		<i>RMSE</i>	0.5653	0.4306	0.2922	0.1847	0.1172	0.5210	0.3868	0.2633	0.1728	0.1213	
	MSOLS-B	<i>Mean</i>	0.4367	0.5726	0.7136	0.8268	0.9044	1.5130	1.3767	1.2484	1.1483	1.0807	
		<i>SD</i>	0.0470	0.0527	0.0580	0.0642	0.0679	0.0907	0.0878	0.0872	0.0887	0.0906	
		<i>RMSE</i>	0.5653	0.4306	0.2922	0.1847	0.1172	0.5210	0.3868	0.2633	0.1728	0.1213	
	<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>		
MSII-A	<i>Mean</i>	1.0218	1.0152	1.0130	1.0080	1.0059	0.9780	0.9848	0.9877	0.9920	0.9934		
	<i>SD</i>	0.1728	0.1290	0.1005	0.0875	0.0802	0.1691	0.1335	0.1121	0.1022	0.0977		
	<i>RMSE</i>	0.1741	0.1299	0.1013	0.0879	0.0804	0.1705	0.1344	0.1128	0.1025	0.0980		
	$\overline{SE}$	0.1789	0.1149	0.0887	0.0783	0.0742	0.1722	0.1218	0.1019	0.0939	0.0907		
	<i>CR</i>	96%	91%	92%	92%	92%	96%	92%	93%	93%	93%		
MSII-B	<i>Mean</i>	1.0218	1.0152	1.0130	1.0080	1.0059	0.9780	0.9847	0.9877	0.9919	0.9934		
	<i>SD</i>	0.1728	0.1290	0.1005	0.0875	0.0802	0.1691	0.1335	0.1121	0.1022	0.0977		
	<i>RMSE</i>	0.1741	0.1299	0.1013	0.0879	0.0804	0.1705	0.1344	0.1128	0.1025	0.0980		
	$\overline{SE}$	0.1789	0.1149	0.0887	0.0783	0.0742	0.1722	0.1218	0.1019	0.0940	0.0907		
	<i>CR</i>	96%	91%	92%	92%	92%	96%	92%	93%	93%	93%		
(2000, 4000) $\implies \kappa = 1/2$	OLS*	<i>Mean</i>	0.9993						1.0016				
		<i>SD</i>	0.0183						0.0410				
		<i>RMSE</i>	0.0183						0.0410				
		$\overline{SE}$	0.0183						0.0404				
		<i>CR</i>	95%						94%				
		<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	
	MSOLS-A	<i>Mean</i>	0.4350	0.5719	0.7112	0.8247	0.9026	1.5185	1.3830	1.2556	1.1540	1.0859	
		<i>SD</i>	0.0398	0.0465	0.0526	0.0577	0.0612	0.0863	0.0833	0.0822	0.0832	0.0842	
		<i>RMSE</i>	0.5664	0.4306	0.2936	0.1845	0.1151	0.5256	0.3919	0.2685	0.1750	0.1203	
	MSOLS-B	<i>Mean</i>	0.4350	0.5719	0.7112	0.8247	0.9026	1.5185	1.3830	1.2556	1.1540	1.0859	
		<i>SD</i>	0.0398	0.0465	0.0526	0.0577	0.0612	0.0863	0.0833	0.0822	0.0832	0.0842	
		<i>RMSE</i>	0.5664	0.4306	0.2936	0.1845	0.1151	0.5256	0.3919	0.2685	0.1750	0.1203	
	<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>		
MSII-A	<i>Mean</i>	1.0184	1.0133	1.0089	1.0053	1.0037	0.9849	0.9919	0.9961	0.9979	0.9988		
	<i>SD</i>	0.1566	0.1144	0.0900	0.0775	0.0715	0.1562	0.1234	0.1036	0.0945	0.0900		
	<i>RMSE</i>	0.1577	0.1152	0.0904	0.0776	0.0716	0.1569	0.1237	0.1037	0.0945	0.0900		
	$\overline{SE}$	0.1623	0.1087	0.0867	0.0777	0.0739	0.1597	0.1177	0.1007	0.0937	0.0906		
	<i>CR</i>	97%	94%	94%	95%	96%	96%	93%	94%	95%	94%		
MSII-B	<i>Mean</i>	1.0184	1.0133	1.0089	1.0053	1.0037	0.9849	0.9919	0.9961	0.9979	0.9988		
	<i>SD</i>	0.1566	0.1144	0.0900	0.0775	0.0715	0.1562	0.1234	0.1036	0.0945	0.0900		
	<i>RMSE</i>	0.1577	0.1152	0.0904	0.0776	0.0716	0.1569	0.1237	0.1037	0.0945	0.0900		
	$\overline{SE}$	0.1623	0.1087	0.0867	0.0777	0.0739	0.1597	0.1177	0.1007	0.0937	0.0906		
	<i>CR</i>	97%	94%	94%	95%	96%	96%	93%	94%	95%	94%		



**Table B1: Continued**

**Model B:**  $g_{22}(z) = 2|z|$

$(n, m)$	Estimator	$\beta_{22}$					$\gamma_1$					
(1000, 500) $\Rightarrow \kappa = 2$	OLS*	Mean	1.0006					1.0002				
		SD	0.0249					0.0535				
		RMSE	0.0250					0.0535				
		$\overline{SE}$	0.0247					0.0530				
		CR	95%					95%				
		<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>
	MSOLS-A	Mean	0.2861	0.4815	0.6709	0.8189	0.9363	1.1709	1.1183	1.0747	1.0411	1.0134
		SD	0.0651	0.0715	0.0830	0.0920	0.1031	0.1184	0.1122	0.1094	0.1105	0.1116
		RMSE	0.7169	0.5234	0.3394	0.2032	0.1212	0.2079	0.1631	0.1325	0.1179	0.1124
	MSOLS-B	Mean	0.2861	0.4815	0.6709	0.8189	0.9363	1.1708	1.1181	1.0746	1.0412	1.0145
		SD	0.0651	0.0715	0.0830	0.0921	0.1031	0.1185	0.1123	0.1095	0.1107	0.1119
		RMSE	0.7169	0.5234	0.3394	0.2031	0.1212	0.2078	0.1630	0.1325	0.1181	0.1129
	<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	
MSII-A	Mean	1.0703	1.0457	1.0356	1.0344	1.0589	0.9881	0.9953	0.9975	0.9961	0.9879	
	SD	0.2721	0.1939	0.1532	0.1308	0.1244	0.1519	0.1294	0.1192	0.1160	0.1146	
	RMSE	0.2811	0.1992	0.1573	0.1353	0.1377	0.1524	0.1295	0.1192	0.1161	0.1153	
	$\overline{SE}$	0.2341	0.1445	0.1130	0.1032	0.1019	0.1310	0.1104	0.1014	0.0974	0.0956	
	CR	94%	89%	87%	89%	88%	91%	91%	91%	91%	90%	
MSII-B	Mean	1.0703	1.0456	1.0356	1.0344	1.0590	0.9878	0.9951	0.9974	0.9962	0.9890	
	SD	0.2721	0.1939	0.1532	0.1308	0.1245	0.1520	0.1294	0.1193	0.1161	0.1150	
	RMSE	0.2811	0.1992	0.1573	0.1353	0.1377	0.1525	0.1295	0.1193	0.1162	0.1155	
	$\overline{SE}$	0.2341	0.1445	0.1130	0.1032	0.1019	0.1311	0.1104	0.1014	0.0974	0.0958	
	CR	94%	89%	87%	89%	89%	91%	91%	90%	91%	90%	
(1000, 1000) $\Rightarrow \kappa = 1$	OLS*	Mean	1.0004					0.9970				
		SD	0.0253					0.0527				
		RMSE	0.0253					0.0528				
		$\overline{SE}$	0.0247					0.0529				
		CR	95%					95%				
		<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>
	MSOLS-A	Mean	0.2821	0.4751	0.6606	0.8039	0.9018	1.1686	1.1156	1.0724	1.0428	1.0208
		SD	0.0575	0.0646	0.0712	0.0777	0.0822	0.1064	0.1011	0.0997	0.0995	0.0992
		RMSE	0.7202	0.5288	0.3468	0.2109	0.1280	0.1994	0.1536	0.1232	0.1083	0.1013
	MSOLS-B	Mean	0.2821	0.4751	0.6606	0.8039	0.9019	1.1685	1.1156	1.0724	1.0428	1.0211
		SD	0.0575	0.0646	0.0712	0.0777	0.0822	0.1065	0.1011	0.0997	0.0996	0.0992
		RMSE	0.7202	0.5288	0.3468	0.2109	0.1280	0.1994	0.1536	0.1232	0.1084	0.1015
	<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	
MSII-A	Mean	1.0391	1.0222	1.0143	1.0115	1.0168	0.9901	0.9942	0.9967	0.9994	0.9969	
	SD	0.2265	0.1625	0.1228	0.1047	0.0963	0.1328	0.1149	0.1067	0.1034	0.1011	
	RMSE	0.2298	0.1641	0.1236	0.1053	0.0977	0.1332	0.1150	0.1068	0.1034	0.1012	
	$\overline{SE}$	0.2006	0.1326	0.1085	0.1003	0.0980	0.1272	0.1095	0.1012	0.0972	0.0954	
	CR	94%	90%	92%	94%	95%	95%	94%	93%	93%	94%	
MSII-B	Mean	1.0391	1.0222	1.0143	1.0116	1.0168	0.9901	0.9942	0.9967	0.9994	0.9972	
	SD	0.2265	0.1625	0.1228	0.1047	0.0963	0.1329	0.1149	0.1068	0.1034	0.1012	
	RMSE	0.2298	0.1641	0.1236	0.1053	0.0977	0.1332	0.1150	0.1068	0.1034	0.1012	
	$\overline{SE}$	0.2006	0.1326	0.1085	0.1003	0.0980	0.1272	0.1095	0.1012	0.0972	0.0954	
	CR	94%	90%	92%	94%	95%	95%	93%	93%	93%	94%	

**Table B1: Continued**

**Model B: Continued**

$(n, m)$	Estimator	$\beta_{22}$					$\gamma_1$					
(1000, 2000) $\Rightarrow \kappa = 1/2$	OLS*	Mean	0.9988					0.9981				
		SD	0.0244					0.0535				
		RMSE	0.0244					0.0536				
		$\overline{SE}$	0.0248					0.0529				
		CR	96%					94%				
		<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>
	MSOLS-A	Mean	0.2809	0.4717	0.6557	0.7963	0.8898	1.1667	1.1163	1.0737	1.0434	1.0229
		SD	0.0540	0.0610	0.0677	0.0728	0.0763	0.1101	0.1034	0.0997	0.0966	0.0968
		RMSE	0.7211	0.5318	0.3509	0.2164	0.1340	0.1998	0.1557	0.1239	0.1059	0.0995
	MSOLS-B	Mean	0.2809	0.4717	0.6557	0.7963	0.8898	1.1666	1.1163	1.0736	1.0434	1.0230
		SD	0.0540	0.0610	0.0677	0.0728	0.0763	0.1101	0.1034	0.0996	0.0966	0.0968
		RMSE	0.7211	0.5318	0.3509	0.2164	0.1340	0.1998	0.1557	0.1239	0.1059	0.0995
	<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	
MSII-A	Mean	1.0310	1.0096	1.0047	1.0006	1.0023	0.9914	0.9984	0.9993	1.0008	0.9996	
	SD	0.2006	0.1402	0.1102	0.0948	0.0875	0.1326	0.1137	0.1045	0.0984	0.0977	
	RMSE	0.2030	0.1405	0.1103	0.0948	0.0876	0.1329	0.1138	0.1045	0.0984	0.0977	
	$\overline{SE}$	0.1838	0.1257	0.1059	0.0988	0.0965	0.1252	0.1088	0.1009	0.0970	0.0952	
	CR	95%	93%	94%	96%	98%	94%	94%	95%	94%	94%	
MSII-B	Mean	1.0310	1.0096	1.0047	1.0006	1.0023	0.9914	0.9984	0.9993	1.0008	0.9997	
	SD	0.2006	0.1402	0.1102	0.0948	0.0875	0.1326	0.1137	0.1045	0.0984	0.0977	
	RMSE	0.2030	0.1405	0.1103	0.0948	0.0876	0.1328	0.1138	0.1045	0.0984	0.0977	
	$\overline{SE}$	0.1838	0.1257	0.1059	0.0988	0.0965	0.1252	0.1088	0.1009	0.0971	0.0952	
	CR	94%	93%	94%	96%	98%	94%	94%	94%	94%	94%	
(2000, 1000) $\Rightarrow \kappa = 2$	OLS*	Mean	0.9992					0.9981				
		SD	0.0169					0.0377				
		RMSE	0.0169					0.0377				
		$\overline{SE}$	0.0175					0.0375				
		CR	96%					95%				
		<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>
	MSOLS-A	Mean	0.2814	0.4738	0.6604	0.8051	0.9047	1.1649	1.1134	1.0708	1.0387	1.0173
		SD	0.0445	0.0503	0.0584	0.0647	0.0706	0.0822	0.0785	0.0778	0.0773	0.0774
		RMSE	0.7200	0.5286	0.3446	0.2054	0.1186	0.1843	0.1379	0.1052	0.0864	0.0794
	MSOLS-B	Mean	0.2814	0.4738	0.6604	0.8051	0.9047	1.1648	1.1134	1.0708	1.0388	1.0176
		SD	0.0445	0.0503	0.0584	0.0647	0.0706	0.0822	0.0785	0.0779	0.0773	0.0775
		RMSE	0.7200	0.5286	0.3446	0.2054	0.1186	0.1842	0.1379	0.1052	0.0865	0.0794
	<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	
MSII-A	Mean	1.0290	1.0161	1.0140	1.0134	1.0203	0.9916	0.9959	0.9969	0.9958	0.9937	
	SD	0.1762	0.1307	0.1065	0.0908	0.0850	0.1036	0.0889	0.0844	0.0806	0.0792	
	RMSE	0.1786	0.1317	0.1074	0.0918	0.0874	0.1039	0.0890	0.0844	0.0807	0.0794	
	$\overline{SE}$	0.1558	0.0990	0.0785	0.0716	0.0697	0.0905	0.0776	0.0717	0.0688	0.0675	
	CR	93%	87%	87%	88%	89%	92%	92%	90%	90%	90%	
MSII-B	Mean	1.0291	1.0161	1.0140	1.0134	1.0202	0.9915	0.9959	0.9969	0.9958	0.9940	
	SD	0.1762	0.1307	0.1065	0.0908	0.0850	0.1036	0.0889	0.0844	0.0806	0.0792	
	RMSE	0.1786	0.1317	0.1074	0.0918	0.0874	0.1039	0.0890	0.0844	0.0807	0.0794	
	$\overline{SE}$	0.1558	0.0990	0.0785	0.0716	0.0697	0.0905	0.0776	0.0717	0.0689	0.0675	
	CR	93%	87%	87%	88%	89%	92%	92%	91%	90%	90%	

**Table B1: Continued**

**Model B: Continued**

$(n, m)$	Estimator	$\beta_{22}$					$\gamma_1$					
(2000, 2000) $\implies \kappa = 1$	OLS*	Mean	1.0004					0.9987				
		SD	0.0172					0.0373				
		RMSE	0.0172					0.0373				
		$\overline{SE}$	0.0175					0.0375				
		CR	95%					94%				
		<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>
	MSOLS-A	Mean	0.2825	0.4758	0.6618	0.8015	0.8945	1.1666	1.1139	1.0711	1.0403	1.0204
		SD	0.0404	0.0462	0.0520	0.0565	0.0613	0.0781	0.0745	0.0722	0.0714	0.0715
		RMSE	0.7186	0.5262	0.3422	0.2064	0.1220	0.1840	0.1360	0.1013	0.0820	0.0743
	MSOLS-B	Mean	0.2825	0.4758	0.6618	0.8015	0.8945	1.1666	1.1139	1.0711	1.0403	1.0205
		SD	0.0404	0.0462	0.0520	0.0565	0.0613	0.0781	0.0745	0.0722	0.0714	0.0715
		RMSE	0.7186	0.5263	0.3422	0.2064	0.1220	0.1840	0.1360	0.1013	0.0820	0.0744
	<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	
MSII-A	Mean	1.0205	1.0133	1.0115	1.0063	1.0070	0.9927	0.9952	0.9965	0.9974	0.9970	
	SD	0.1477	0.1132	0.0890	0.0760	0.0720	0.0932	0.0813	0.0760	0.0734	0.0726	
	RMSE	0.1491	0.1140	0.0898	0.0763	0.0723	0.0935	0.0815	0.0761	0.0734	0.0726	
	$\overline{SE}$	0.1366	0.0921	0.0759	0.0702	0.0683	0.0887	0.0771	0.0714	0.0687	0.0673	
	CR	94%	90%	91%	92%	93%	94%	93%	94%	93%	93%	
MSII-B	Mean	1.0205	1.0133	1.0115	1.0063	1.0070	0.9927	0.9952	0.9965	0.9974	0.9971	
	SD	0.1477	0.1132	0.0890	0.0760	0.0720	0.0932	0.0813	0.0760	0.0734	0.0726	
	RMSE	0.1491	0.1140	0.0898	0.0763	0.0723	0.0935	0.0815	0.0761	0.0734	0.0726	
	$\overline{SE}$	0.1366	0.0921	0.0759	0.0702	0.0683	0.0887	0.0771	0.0714	0.0687	0.0674	
	CR	94%	90%	91%	92%	93%	94%	93%	94%	93%	93%	
(2000, 4000) $\implies \kappa = 1/2$	OLS*	Mean	0.9991					1.0011				
		SD	0.0177					0.0379				
		RMSE	0.0178					0.0379				
		$\overline{SE}$	0.0175					0.0374				
		CR	94%					94%				
		<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>
	MSOLS-A	Mean	0.2828	0.4757	0.6594	0.7997	0.8916	1.1686	1.1178	1.0754	1.0439	1.0242
		SD	0.0383	0.0431	0.0480	0.0514	0.0536	0.0754	0.0706	0.0673	0.0662	0.0654
		RMSE	0.7182	0.5261	0.3440	0.2068	0.1209	0.1847	0.1373	0.1010	0.0794	0.0697
	MSOLS-B	Mean	0.2828	0.4757	0.6594	0.7997	0.8916	1.1686	1.1178	1.0754	1.0439	1.0243
		SD	0.0383	0.0431	0.0480	0.0514	0.0536	0.0754	0.0706	0.0673	0.0662	0.0654
		RMSE	0.7182	0.5261	0.3440	0.2068	0.1209	0.1847	0.1373	0.1010	0.0794	0.0697
	<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	
MSII-A	Mean	1.0154	1.0098	1.0061	1.0029	1.0032	0.9978	1.0012	1.0022	1.0016	1.0013	
	SD	0.1335	0.0944	0.0758	0.0660	0.0610	0.0893	0.0772	0.0702	0.0676	0.0660	
	RMSE	0.1344	0.0949	0.0761	0.0660	0.0611	0.0893	0.0772	0.0703	0.0676	0.0660	
	$\overline{SE}$	0.1260	0.0882	0.0745	0.0696	0.0679	0.0875	0.0767	0.0713	0.0686	0.0673	
	CR	93%	94%	95%	96%	97%	95%	95%	96%	95%	95%	
MSII-B	Mean	1.0154	1.0098	1.0061	1.0029	1.0032	0.9978	1.0012	1.0022	1.0016	1.0013	
	SD	0.1335	0.0944	0.0758	0.0660	0.0610	0.0893	0.0772	0.0702	0.0676	0.0660	
	RMSE	0.1344	0.0949	0.0761	0.0660	0.0611	0.0893	0.0772	0.0703	0.0676	0.0660	
	$\overline{SE}$	0.1260	0.0882	0.0745	0.0696	0.0679	0.0875	0.0767	0.0713	0.0686	0.0673	
	CR	93%	94%	95%	96%	97%	95%	95%	96%	95%	95%	

**Table B1: Continued**

**Model C:**  $g_{22}(z) = 4\sqrt{|z/2|(1-|z/2|)}\sin\{2\pi(1+\epsilon)/(|z/2|+\epsilon)\}$ ,  $\epsilon = 0.05$

$(n, m)$	Estimator	$\beta_{22}$					$\gamma_1$					
(1000, 500) $\Rightarrow \kappa = 2$	OLS*	<i>Mean</i>	1.0003					1.0003				
		<i>SD</i>	0.0209					0.0531				
		<i>RMSE</i>	0.0209					0.0531				
		$\overline{SE}$	0.0207					0.0528				
		<i>CR</i>	95%					95%				
		<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>
	MSOLS-A	<i>Mean</i>	0.5493	0.7088	0.8356	0.9287	1.0171	1.0547	1.0316	1.0192	1.0099	1.0001
		<i>SD</i>	0.0604	0.0634	0.0707	0.0777	0.0901	0.1261	0.1178	0.1128	0.1131	0.1150
		<i>RMSE</i>	0.4547	0.2980	0.1790	0.1055	0.0917	0.1375	0.1220	0.1145	0.1135	0.1150
	MSOLS-B	<i>Mean</i>	0.5493	0.7088	0.8356	0.9287	1.0172	1.0545	1.0314	1.0191	1.0100	1.0012
		<i>SD</i>	0.0604	0.0634	0.0707	0.0777	0.0901	0.1262	0.1178	0.1129	0.1132	0.1154
		<i>RMSE</i>	0.4547	0.2980	0.1790	0.1054	0.0917	0.1375	0.1219	0.1145	0.1137	0.1154
(1000, 1000) $\Rightarrow \kappa = 1$	OLS*	<i>Mean</i>	1.0003					0.9970				
		<i>SD</i>	0.0202					0.0529				
		<i>RMSE</i>	0.0202					0.0529				
		$\overline{SE}$	0.0207					0.0527				
		<i>CR</i>	96%					95%				
		<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>
	MSOLS-A	<i>Mean</i>	0.5556	0.7148	0.8355	0.9203	0.9854	1.0513	1.0272	1.0145	1.0091	1.0029
		<i>SD</i>	0.0512	0.0546	0.0582	0.0611	0.0658	0.1134	0.1052	0.1019	0.1008	0.1002
		<i>RMSE</i>	0.4474	0.2903	0.1745	0.1004	0.0674	0.1245	0.1087	0.1029	0.1012	0.1003
	MSOLS-B	<i>Mean</i>	0.5556	0.7148	0.8355	0.9203	0.9854	1.0513	1.0271	1.0145	1.0092	1.0033
		<i>SD</i>	0.0512	0.0546	0.0582	0.0611	0.0658	0.1135	0.1052	0.1020	0.1008	0.1003
		<i>RMSE</i>	0.4474	0.2903	0.1745	0.1005	0.0674	0.1245	0.1087	0.1030	0.1012	0.1004
(1000, 1000) $\Rightarrow \kappa = 1$	OLS*	<i>Mean</i>	1.0003					0.9970				
		<i>SD</i>	0.0202					0.0529				
		<i>RMSE</i>	0.0202					0.0529				
		$\overline{SE}$	0.0207					0.0527				
		<i>CR</i>	96%					95%				
		<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>
	MSII-A	<i>Mean</i>	1.0251	1.0142	1.0126	1.0221	1.0458	0.9970	0.9980	0.9993	1.0013	0.9988
		<i>SD</i>	0.1141	0.0906	0.0774	0.0711	0.0718	0.1231	0.1098	0.1040	0.1019	0.1008
		<i>RMSE</i>	0.1168	0.0917	0.0784	0.0744	0.0852	0.1231	0.1098	0.1040	0.1019	0.1008
	MSII-B	<i>Mean</i>	1.0251	1.0142	1.0126	1.0221	1.0458	0.9970	0.9979	0.9993	1.0013	0.9991
		<i>SD</i>	0.1141	0.0906	0.0774	0.0711	0.0718	0.1231	0.1098	0.1040	0.1019	0.1009
		<i>RMSE</i>	0.1168	0.0917	0.0784	0.0744	0.0852	0.1231	0.1099	0.1040	0.1019	0.1009
MSII-A	<i>Mean</i>	1.0251	1.0142	1.0126	1.0221	1.0458	0.9970	0.9980	0.9993	1.0013	0.9988	
	<i>SD</i>	0.1141	0.0906	0.0774	0.0711	0.0718	0.1231	0.1098	0.1040	0.1019	0.1008	
	<i>RMSE</i>	0.1168	0.0917	0.0784	0.0744	0.0852	0.1231	0.1098	0.1040	0.1019	0.1008	
MSII-B	<i>Mean</i>	1.0251	1.0142	1.0126	1.0221	1.0458	0.9970	0.9979	0.9993	1.0013	0.9991	
	<i>SD</i>	0.1141	0.0906	0.0774	0.0711	0.0718	0.1231	0.1098	0.1040	0.1019	0.1009	
	<i>RMSE</i>	0.1168	0.0917	0.0784	0.0744	0.0852	0.1231	0.1099	0.1040	0.1019	0.1009	
MSII-A	<i>Mean</i>	1.0251	1.0142	1.0126	1.0221	1.0458	0.9970	0.9980	0.9993	1.0013	0.9988	
	<i>SD</i>	0.1141	0.0906	0.0774	0.0711	0.0718	0.1231	0.1098	0.1040	0.1019	0.1008	
	<i>RMSE</i>	0.1168	0.0917	0.0784	0.0744	0.0852	0.1231	0.1098	0.1040	0.1019	0.1008	
MSII-B	<i>Mean</i>	1.0251	1.0142	1.0126	1.0221	1.0458	0.9970	0.9979	0.9993	1.0013	0.9991	
	<i>SD</i>	0.1141	0.0906	0.0774	0.0711	0.0718	0.1231	0.1098	0.1040	0.1019	0.1009	
	<i>RMSE</i>	0.1168	0.0917	0.0784	0.0744	0.0852	0.1231	0.1099	0.1040	0.1019	0.1009	

**Table B1: Continued**

**Model C: Continued**

$(n, m)$	Estimator	$\beta_{22}$					$\gamma_1$					
(1000, 2000) $\Rightarrow \kappa = 1/2$	OLS*	Mean	1.0009					0.9978				
		SD	0.0210					0.0533				
		RMSE	0.0210					0.0533				
		$\overline{SE}$	0.0207					0.0528				
		CR	95%					94%				
		<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>
	MSOLS-A	Mean	0.5590	0.7188	0.8391	0.9188	0.9732	1.0495	1.0280	1.0145	1.0075	1.0023
		SD	0.0487	0.0497	0.0514	0.0533	0.0563	0.1169	0.1069	0.1016	0.0981	0.0985
		RMSE	0.4437	0.2856	0.1690	0.0971	0.0623	0.1270	0.1106	0.1026	0.0984	0.0985
	MSOLS-B	Mean	0.5590	0.7188	0.8391	0.9188	0.9732	1.0495	1.0280	1.0145	1.0075	1.0024
		SD	0.0487	0.0497	0.0514	0.0533	0.0563	0.1169	0.1070	0.1016	0.0981	0.0985
		RMSE	0.4437	0.2856	0.1690	0.0971	0.0623	0.1269	0.1106	0.1026	0.0984	0.0985
	<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	
MSII-A	Mean	1.0160	1.0090	1.0090	1.0136	1.0260	0.9966	1.0002	0.9997	0.9999	0.9984	
	SD	0.0996	0.0762	0.0650	0.0602	0.0601	0.1261	0.1101	0.1027	0.0986	0.0987	
	RMSE	0.1008	0.0767	0.0656	0.0617	0.0654	0.1261	0.1101	0.1027	0.0986	0.0988	
	$\overline{SE}$	0.0943	0.0701	0.0611	0.0582	0.0586	0.1191	0.1061	0.0991	0.0956	0.0941	
	CR	93%	93%	94%	94%	93%	94%	94%	94%	94%	94%	
MSII-B	Mean	1.0160	1.0090	1.0090	1.0136	1.0260	0.9966	1.0002	0.9997	0.9999	0.9985	
	SD	0.0996	0.0762	0.0650	0.0602	0.0601	0.1260	0.1101	0.1027	0.0986	0.0988	
	RMSE	0.1008	0.0767	0.0656	0.0617	0.0654	0.1261	0.1101	0.1027	0.0986	0.0988	
	$\overline{SE}$	0.0943	0.0701	0.0611	0.0582	0.0586	0.1191	0.1061	0.0991	0.0956	0.0942	
	CR	93%	93%	94%	94%	93%	94%	94%	94%	94%	94%	
(2000, 1000) $\Rightarrow \kappa = 2$	OLS*	Mean	0.9996					0.9979				
		SD	0.0138					0.0375				
		RMSE	0.0138					0.0375				
		$\overline{SE}$	0.0146					0.0374				
		CR	95%					95%				
		<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>
	MSOLS-A	Mean	0.5561	0.7146	0.8361	0.9204	0.9865	1.0501	1.0272	1.0142	1.0055	1.0000
		SD	0.0413	0.0416	0.0453	0.0487	0.0531	0.0880	0.0816	0.0801	0.0791	0.0797
		RMSE	0.4458	0.2884	0.1701	0.0933	0.0547	0.1013	0.0860	0.0813	0.0793	0.0797
	MSOLS-B	Mean	0.5561	0.7146	0.8361	0.9204	0.9865	1.0501	1.0272	1.0142	1.0056	1.0003
		SD	0.0413	0.0416	0.0453	0.0487	0.0531	0.0880	0.0816	0.0801	0.0791	0.0797
		RMSE	0.4458	0.2884	0.1701	0.0933	0.0547	0.1012	0.0860	0.0813	0.0793	0.0797
	<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	
MSII-A	Mean	1.0256	1.0129	1.0128	1.0221	1.0467	0.9976	0.9994	0.9997	0.9978	0.9959	
	SD	0.0926	0.0716	0.0617	0.0576	0.0584	0.0991	0.0860	0.0825	0.0804	0.0805	
	RMSE	0.0961	0.0728	0.0631	0.0617	0.0748	0.0991	0.0860	0.0825	0.0804	0.0806	
	$\overline{SE}$	0.0824	0.0554	0.0457	0.0433	0.0449	0.0851	0.0754	0.0705	0.0681	0.0673	
	CR	87%	81%	77%	76%	66%	86%	87%	84%	83%	83%	
MSII-B	Mean	1.0256	1.0129	1.0128	1.0221	1.0468	0.9975	0.9994	0.9997	0.9979	0.9962	
	SD	0.0926	0.0716	0.0617	0.0576	0.0584	0.0991	0.0860	0.0826	0.0804	0.0805	
	RMSE	0.0961	0.0728	0.0631	0.0617	0.0748	0.0991	0.0860	0.0826	0.0804	0.0806	
	$\overline{SE}$	0.0824	0.0554	0.0457	0.0433	0.0449	0.0851	0.0754	0.0705	0.0681	0.0674	
	CR	87%	81%	77%	76%	66%	86%	87%	84%	83%	83%	

**Table B1: Continued**

**Model C: Continued**

$(n, m)$	Estimator	$\beta_{22}$					$\gamma_1$					
(2000, 2000) $\Rightarrow \kappa = 1$	OLS*	Mean	0.9995					0.9988				
		SD	0.0145					0.0372				
		RMSE	0.0145					0.0372				
		$\overline{SE}$	0.0147					0.0374				
		CR	96%					94%				
			<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>
	MSOLS-A	Mean	0.5602	0.7204	0.8406	0.9191	0.9730	1.0502	1.0263	1.0137	1.0063	1.0019
		SD	0.0359	0.0380	0.0399	0.0416	0.0437	0.0814	0.0758	0.0729	0.0716	0.0719
		RMSE	0.4413	0.2821	0.1643	0.0909	0.0514	0.0956	0.0803	0.0742	0.0719	0.0719
	MSOLS-B	Mean	0.5602	0.7204	0.8406	0.9191	0.9729	1.0502	1.0263	1.0137	1.0063	1.0020
		SD	0.0359	0.0380	0.0399	0.0416	0.0437	0.0814	0.0758	0.0729	0.0716	0.0719
		RMSE	0.4413	0.2821	0.1643	0.0909	0.0514	0.0956	0.0803	0.0742	0.0719	0.0719
(2000, 4000) $\Rightarrow \kappa = 1/2$	OLS*	Mean	0.9995					1.0010				
		SD	0.0146					0.0377				
		RMSE	0.0146					0.0377				
		$\overline{SE}$	0.0146					0.0373				
		CR	95%					94%				
			<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>
	MSOLS-A	Mean	0.5615	0.7217	0.8404	0.9168	0.9651	1.0535	1.0312	1.0181	1.0097	1.0056
		SD	0.0327	0.0345	0.0366	0.0381	0.0390	0.0792	0.0727	0.0683	0.0667	0.0655
		RMSE	0.4397	0.2804	0.1637	0.0915	0.0524	0.0956	0.0791	0.0706	0.0674	0.0658
	MSOLS-B	Mean	0.5615	0.7217	0.8404	0.9168	0.9651	1.0535	1.0312	1.0181	1.0097	1.0056
		SD	0.0327	0.0345	0.0366	0.0381	0.0390	0.0792	0.0727	0.0683	0.0667	0.0655
		RMSE	0.4397	0.2804	0.1637	0.0915	0.0524	0.0956	0.0791	0.0706	0.0674	0.0658
(2000, 4000) $\Rightarrow \kappa = 1/2$	MSII-A	Mean	1.0144	1.0100	1.0099	1.0135	1.0253	0.9961	0.9975	0.9986	0.9985	0.9979
		SD	0.0745	0.0614	0.0519	0.0478	0.0471	0.0879	0.0790	0.0744	0.0724	0.0723
		RMSE	0.0758	0.0622	0.0528	0.0496	0.0535	0.0880	0.0790	0.0745	0.0724	0.0723
		$\overline{SE}$	0.0712	0.0512	0.0436	0.0413	0.0414	0.0843	0.0750	0.0700	0.0676	0.0666
		CR	94%	90%	90%	91%	88%	94%	94%	94%	93%	93%
			<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>
	MSII-B	Mean	1.0144	1.0100	1.0099	1.0135	1.0253	0.9961	0.9975	0.9987	0.9985	0.9979
		SD	0.0745	0.0614	0.0519	0.0478	0.0471	0.0879	0.0790	0.0744	0.0724	0.0723
		RMSE	0.0758	0.0622	0.0528	0.0496	0.0535	0.0880	0.0790	0.0745	0.0724	0.0723
	MSII-B	$\overline{SE}$	0.0712	0.0512	0.0436	0.0413	0.0414	0.0843	0.0750	0.0700	0.0676	0.0666
		CR	94%	90%	90%	91%	88%	95%	94%	94%	93%	93%
		Mean	1.0077	1.0055	1.0056	1.0075	1.0139	1.0001	1.0028	1.0033	1.0021	1.0017
SD		0.0662	0.0521	0.0459	0.0429	0.0415	0.0865	0.0756	0.0694	0.0672	0.0657	
RMSE		0.0667	0.0524	0.0462	0.0435	0.0437	0.0865	0.0757	0.0695	0.0672	0.0658	
$\overline{SE}$		0.0655	0.0490	0.0426	0.0403	0.0398	0.0839	0.0748	0.0699	0.0674	0.0663	
MSII-B	CR	95%	93%	92%	93%	93%	95%	95%	96%	95%	95%	
	Mean	1.0077	1.0055	1.0056	1.0075	1.0139	1.0001	1.0028	1.0033	1.0021	1.0017	
	SD	0.0662	0.0521	0.0459	0.0429	0.0415	0.0865	0.0756	0.0694	0.0672	0.0657	
	RMSE	0.0667	0.0524	0.0462	0.0435	0.0437	0.0865	0.0757	0.0695	0.0672	0.0658	
	$\overline{SE}$	0.0655	0.0490	0.0426	0.0403	0.0398	0.0839	0.0748	0.0699	0.0674	0.0663	
	CR	95%	93%	92%	93%	93%	95%	95%	96%	95%	95%	

**Table B2:** Monte Carlo Results ( $d_3 = 2$ )

**Model A:**  $g_{22}(z) = z + (5/\tau)\phi(z/\tau)$ ,  $\tau = 0.75$

$(n, m)$	Estimator	$\beta_{22}$					$\gamma_1$					
(1000, 500) $\Rightarrow \kappa = 2$	OLS*	Mean	0.9995					0.9999				
		SD	0.0205					0.0624				
		RMSE	0.0205					0.0624				
		$\overline{SE}$	0.0215					0.0611				
		CR	96%					94%				
		<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>
MSOLS-A	Mean	0.6988	0.7530	0.7082	0.5053	0.0499	1.3006	1.2217	1.2213	1.3284	1.5948	
	SD	0.0800	0.0866	0.1020	0.1374	0.2059	0.2182	0.2148	0.2413	0.2960	0.4040	
	RMSE	0.3116	0.2618	0.3091	0.5134	0.9722	0.3715	0.3087	0.3274	0.4421	0.7190	
MSOLS-B	Mean	0.7126	0.7687	0.7266	0.5281	0.0821	1.1556	1.1029	1.1112	1.2120	1.4526	
	SD	0.0808	0.0872	0.1026	0.1385	0.2086	0.2256	0.2183	0.2440	0.3037	0.4254	
	RMSE	0.2985	0.2472	0.2921	0.4918	0.9413	0.2741	0.2413	0.2682	0.3704	0.6211	
MSII-FM-A	Poly.	<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		
	Mean	0.8370	0.8991	0.8973	0.9023		1.0865	1.0279	1.0302	1.0207		
	SD	0.2221	0.2277	0.2283	0.2196		0.2762	0.2873	0.2901	0.2798		
	RMSE	0.2755	0.2490	0.2503	0.2404		0.2894	0.2887	0.2917	0.2806		
	$\overline{SE}$	–	0.3053	0.3055	0.3058		–	0.3055	0.3061	0.3051		
MSII-FM-B	Mean	0.8706	0.9002	0.8985	0.9031		0.9249	1.0216	1.0235	1.0151		
	SD	0.2254	0.2257	0.2262	0.2182		0.2837	0.2914	0.2952	0.2828		
	RMSE	0.2599	0.2468	0.2479	0.2388		0.2934	0.2922	0.2962	0.2832		
	$\overline{SE}$	–	0.3093	0.3094	0.3098		–	0.3088	0.3094	0.3081		
	CR	–	99%	99%	99%		–	96%	96%	96%		
(1000, 1000) $\Rightarrow \kappa = 1$	OLS*	Mean	0.9996					0.9979				
		SD	0.0220					0.0591				
		RMSE	0.0220					0.0592				
		$\overline{SE}$	0.0215					0.0613				
		CR	95%					95%				
		<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>
MSOLS-A	Mean	0.6607	0.7490	0.7678	0.6807	0.4267	1.3184	1.2184	1.1868	1.2287	1.3823	
	SD	0.0632	0.0664	0.0743	0.0867	0.1124	0.1736	0.1617	0.1618	0.1776	0.2231	
	RMSE	0.3451	0.2597	0.2438	0.3308	0.5842	0.3627	0.2717	0.2472	0.2895	0.4427	
MSOLS-B	Mean	0.6679	0.7568	0.7766	0.6922	0.4428	1.2411	1.1566	1.1317	1.1691	1.3084	
	SD	0.0633	0.0667	0.0743	0.0868	0.1128	0.1746	0.1617	0.1629	0.1789	0.2296	
	RMSE	0.3381	0.2522	0.2354	0.3198	0.5685	0.2976	0.2251	0.2094	0.2462	0.3845	
MSII-FM-A	Poly.	<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		
	Mean	0.8982	0.9252	0.9246	0.9313		1.0614	1.0363	1.0369	1.0240		
	SD	0.1497	0.1509	0.1511	0.1491		0.2071	0.2108	0.2121	0.2131		
	RMSE	0.1810	0.1685	0.1688	0.1642		0.2160	0.2139	0.2153	0.2145		
	$\overline{SE}$	–	0.1879	0.1880	0.1878		–	0.2200	0.2202	0.2196		
MSII-FM-B	Mean	0.9152	0.9259	0.9252	0.9323		0.9754	1.0326	1.0335	1.0200		
	SD	0.1513	0.1507	0.1509	0.1489		0.2095	0.2128	0.2143	0.2141		
	RMSE	0.1734	0.1679	0.1684	0.1635		0.2109	0.2153	0.2169	0.2151		
	$\overline{SE}$	–	0.1891	0.1891	0.1890		–	0.2207	0.2209	0.2202		
	CR	–	92%	92%	94%		–	91%	91%	91%		

**Table B2: Continued**

**Model A: Continued**

$(n, m)$	Estimator		$\beta_{22}$					$\gamma_1$				
(1000, 2000) $\Rightarrow \kappa = 1/2$	OLS*	Mean	0.9995					1.0014				
		SD	0.0215					0.0611				
		RMSE	0.0215					0.0612				
		$\overline{SE}$	0.0215					0.0612				
		CR	96%					96%				
		<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>
	MSOLS-A	Mean	0.6406	0.7496	0.8068	0.7890	0.6660	1.3207	1.2132	1.1559	1.1546	1.2254
		SD	0.0552	0.0585	0.0611	0.0652	0.0730	0.1517	0.1363	0.1323	0.1375	0.1517
		RMSE	0.3636	0.2571	0.2026	0.2208	0.3419	0.3548	0.2531	0.2045	0.2069	0.2717
	MSOLS-B	Mean	0.6443	0.7536	0.8113	0.7945	0.6737	1.2821	1.1824	1.1279	1.1253	1.1889
		SD	0.0554	0.0587	0.0612	0.0653	0.0731	0.1517	0.1367	0.1330	0.1389	0.1536
RMSE		0.3600	0.2533	0.1983	0.2156	0.3344	0.3203	0.2279	0.1845	0.1871	0.2435	
	<i>Poly.</i>	<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		
MSII-FM-A	Mean	0.9382	0.9508	0.9506	0.9591		1.0328	1.0210	1.0212	1.0111		
	SD	0.1249	0.1245	0.1245	0.1249		0.1819	0.1841	0.1843	0.1831		
	RMSE	0.1393	0.1339	0.1339	0.1315		0.1848	0.1853	0.1855	0.1835		
	$\overline{SE}$	–	0.1367	0.1367	0.1367		–	0.1832	0.1833	0.1831		
	CR	–	95%	95%	95%		–	94%	95%	95%		
MSII-FM-B	Mean	0.9466	0.9509	0.9507	0.9593		0.9901	1.0206	1.0207	1.0103		
	SD	0.1255	0.1241	0.1241	0.1245		0.1819	0.1844	0.1848	0.1827		
	RMSE	0.1364	0.1335	0.1336	0.1310		0.1822	0.1856	0.1859	0.1830		
	$\overline{SE}$	–	0.1371	0.1371	0.1370		–	0.1833	0.1833	0.1831		
	CR	–	95%	94%	95%		–	95%	94%	95%		
(2000, 1000) $\Rightarrow \kappa = 2$	OLS*	Mean	0.9995					1.0013				
		SD	0.0152					0.0417				
		RMSE	0.0152					0.0417				
		$\overline{SE}$	0.0152					0.0433				
		CR	96%					96%				
		<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>
	MSOLS-A	Mean	0.6609	0.7481	0.7672	0.6777	0.4204	1.3213	1.2244	1.1872	1.2311	1.3861
		SD	0.0496	0.0554	0.0634	0.0760	0.0988	0.1335	0.1295	0.1356	0.1583	0.2075
		RMSE	0.3427	0.2579	0.2412	0.3311	0.5879	0.3480	0.2591	0.2311	0.2801	0.4383
	MSOLS-B	Mean	0.6678	0.7557	0.7759	0.6886	0.4358	1.2464	1.1646	1.1326	1.1728	1.3138
		SD	0.0502	0.0561	0.0642	0.0767	0.0993	0.1357	0.1321	0.1393	0.1627	0.2149
RMSE		0.3360	0.2507	0.2331	0.3207	0.5729	0.2812	0.2111	0.1923	0.2374	0.3803	
	<i>Poly.</i>	<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		
MSII-FM-A	Mean	0.8954	0.9228	0.9221	0.9297		1.0657	1.0383	1.0390	1.0306		
	SD	0.1179	0.1194	0.1197	0.1202		0.1624	0.1650	0.1658	0.1667		
	RMSE	0.1576	0.1422	0.1428	0.1392		0.1751	0.1694	0.1703	0.1695		
	$\overline{SE}$	–	0.1685	0.1685	0.1687		–	0.1767	0.1769	0.1765		
	CR	–	98%	98%	98%		–	96%	96%	96%		
MSII-FM-B	Mean	0.9117	0.9230	0.9224	0.9299		0.9828	1.0374	1.0375	1.0295		
	SD	0.1198	0.1196	0.1197	0.1202		0.1654	0.1662	0.1670	0.1667		
	RMSE	0.1488	0.1423	0.1427	0.1391		0.1663	0.1704	0.1711	0.1693		
	$\overline{SE}$	–	0.1695	0.1696	0.1698		–	0.1775	0.1776	0.1772		
	CR	–	98%	98%	98%		–	96%	95%	96%		



Table B2: *Continued*

Model A: *Continued*

$(n, m)$	Estimator		$\beta_{22}$					$\gamma_1$				
(2000, 2000) $\Rightarrow \kappa = 1$	OLS*	<i>Mean</i>	0.9997					1.0013				
		<i>SD</i>	0.0152					0.0425				
		<i>RMSE</i>	0.0152					0.0425				
		$\overline{SE}$	0.0152					0.0433				
		<i>CR</i>	95%					95%				
		<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>
	MSOLS-A	<i>Mean</i>	0.6419	0.7501	0.8071	0.7874	0.6640	1.3198	1.2134	1.1506	1.1512	1.2243
		<i>SD</i>	0.0430	0.0438	0.0485	0.0540	0.0637	0.1089	0.1018	0.1007	0.1055	0.1193
		<i>RMSE</i>	0.3606	0.2537	0.1989	0.2194	0.3420	0.3379	0.2364	0.1812	0.1844	0.2541
	MSOLS-B	<i>Mean</i>	0.6455	0.7540	0.8116	0.7928	0.6717	1.2805	1.1825	1.1227	1.1222	1.1879
		<i>SD</i>	0.0432	0.0441	0.0486	0.0541	0.0639	0.1100	0.1026	0.1014	0.1065	0.1219
<i>RMSE</i>		0.3571	0.2499	0.1946	0.2141	0.3344	0.3013	0.2094	0.1592	0.1621	0.2239	
	<i>Poly.</i>	<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		
MSII-FM-A	<i>Mean</i>	0.9376	0.9498	0.9496	0.9571		1.0335	1.0218	1.0218	1.0129		
	<i>SD</i>	0.0960	0.0970	0.0970	0.0968		0.1301	0.1310	0.1310	0.1313		
	<i>RMSE</i>	0.1145	0.1092	0.1093	0.1059		0.1344	0.1328	0.1328	0.1320		
	$\overline{SE}$	–	0.1140	0.1140	0.1140		–	0.1387	0.1388	0.1388		
	<i>CR</i>	–	91%	91%	92%		–	92%	92%	93%		
MSII-FM-B	<i>Mean</i>	0.9460	0.9501	0.9498	0.9572		0.9900	1.0207	1.0207	1.0122		
	<i>SD</i>	0.0966	0.0968	0.0968	0.0967		0.1315	0.1319	0.1319	0.1328		
	<i>RMSE</i>	0.1107	0.1089	0.1090	0.1057		0.1319	0.1335	0.1335	0.1334		
	$\overline{SE}$	–	0.1143	0.1143	0.1144		–	0.1389	0.1389	0.1389		
	<i>CR</i>	–	91%	91%	92%		–	92%	92%	93%		
(2000, 4000) $\Rightarrow \kappa = 1/2$	OLS*	<i>Mean</i>	1.0001					1.0002				
		<i>SD</i>	0.0155					0.0424				
		<i>RMSE</i>	0.0155					0.0424				
		$\overline{SE}$	0.0152					0.0433				
		<i>CR</i>	94%					95%				
		<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>
	MSOLS-A	<i>Mean</i>	0.6329	0.7495	0.8281	0.8499	0.8044	1.3248	1.2153	1.1418	1.1153	1.1372
		<i>SD</i>	0.0382	0.0394	0.0418	0.0440	0.0475	0.0979	0.0915	0.0892	0.0898	0.0944
		<i>RMSE</i>	0.3691	0.2536	0.1769	0.1564	0.2013	0.3392	0.2340	0.1675	0.1461	0.1666
	MSOLS-B	<i>Mean</i>	0.6347	0.7514	0.8303	0.8526	0.8079	1.3054	1.1999	1.1279	1.1012	1.1203
		<i>SD</i>	0.0383	0.0394	0.0419	0.0441	0.0476	0.0985	0.0918	0.0894	0.0902	0.0951
<i>RMSE</i>		0.3673	0.2517	0.1748	0.1539	0.1979	0.3209	0.2200	0.1561	0.1356	0.1533	
	<i>Poly.</i>	<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		
MSII-FM-A	<i>Mean</i>	0.9664	0.9715	0.9715	0.9773		1.0183	1.0125	1.0126	1.0073		
	<i>SD</i>	0.0812	0.0815	0.0815	0.0817		0.1177	0.1178	0.1178	0.1199		
	<i>RMSE</i>	0.0878	0.0863	0.0863	0.0848		0.1191	0.1185	0.1185	0.1201		
	$\overline{SE}$	–	0.0887	0.0887	0.0888		–	0.1220	0.1220	0.1222		
	<i>CR</i>	–	95%	95%	95%		–	95%	95%	95%		
MSII-FM-B	<i>Mean</i>	0.9706	0.9715	0.9715	0.9774		0.9968	1.0124	1.0125	1.0070		
	<i>SD</i>	0.0814	0.0815	0.0815	0.0817		0.1186	0.1187	0.1187	0.1204		
	<i>RMSE</i>	0.0866	0.0864	0.0864	0.0848		0.1187	0.1194	0.1193	0.1206		
	$\overline{SE}$	–	0.0889	0.0889	0.0889		–	0.1220	0.1221	0.1222		
	<i>CR</i>	–	95%	95%	95%		–	95%	95%	95%		

Table B2: *Continued*

Model B:  $g_{22}(z) = 2|z|$

$(n, m)$	Estimator		$\beta_{22}$					$\gamma_1$				
(1000, 500) $\Rightarrow \kappa = 2$	OLS*	Mean	1.0003					0.9994				
		SD	0.0203					0.0599				
		RMSE	0.0203					0.0599				
		$\overline{SE}$	0.0196					0.0587				
		CR	93%					95%				
		<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>
	MSOLS-A	Mean	0.4612	0.6874	0.9167	1.1884	1.5820	1.1596	1.0848	1.0062	0.9024	0.7455
		SD	0.0609	0.0674	0.0825	0.1064	0.1546	0.1855	0.1875	0.2116	0.2629	0.3553
		RMSE	0.5422	0.3198	0.1173	0.2164	0.6022	0.2447	0.2058	0.2117	0.2804	0.4371
	MSOLS-B	Mean	0.4627	0.6890	0.9188	1.1918	1.5891	1.0713	1.0151	0.9436	0.8366	0.6582
		SD	0.0611	0.0677	0.0828	0.1070	0.1558	0.1893	0.1893	0.2137	0.2697	0.3748
RMSE		0.5408	0.3183	0.1159	0.2196	0.6093	0.2023	0.1899	0.2210	0.3153	0.5073	
	<i>Poly.</i>	<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		
MSII-FM-A	Mean	1.1865	1.1771	1.1783	1.1989		0.9219	0.9191	0.9197	0.9106		
	SD	0.1697	0.1706	0.1710	0.1751		0.2308	0.2355	0.2373	0.2413		
	RMSE	0.2521	0.2459	0.2471	0.2650		0.2437	0.2490	0.2505	0.2574		
	$\overline{SE}$	–	0.1793	0.1795	0.1817		–	0.1853	0.1860	0.1881		
	CR	–	91%	90%	88%		–	87%	87%	85%		
MSII-FM-B	Mean	1.1902	1.1771	1.1783	1.1987		0.8325	0.9161	0.9164	0.9083		
	SD	0.1703	0.1705	0.1709	0.1750		0.2348	0.2419	0.2446	0.2454		
	RMSE	0.2553	0.2459	0.2470	0.2648		0.2885	0.2560	0.2585	0.2620		
	$\overline{SE}$	–	0.1795	0.1796	0.1819		–	0.1852	0.1860	0.1876		
	CR	–	91%	90%	88%		–	85%	85%	84%		
(1000, 1000) $\Rightarrow \kappa = 1$	OLS*	Mean	0.9996					0.9976				
		SD	0.0194					0.0574				
		RMSE	0.0194					0.0574				
		$\overline{SE}$	0.0196					0.0589				
		CR	94%					96%				
		<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>
	MSOLS-A	Mean	0.4975	0.7009	0.8843	1.0728	1.3237	1.1423	1.0768	1.0249	0.9607	0.8665
		SD	0.0516	0.0552	0.0604	0.0697	0.0873	0.1512	0.1451	0.1465	0.1624	0.2017
		RMSE	0.5052	0.3041	0.1305	0.1007	0.3353	0.2076	0.1642	0.1486	0.1671	0.2419
	MSOLS-B	Mean	0.4984	0.7019	0.8853	1.0741	1.3262	1.0942	1.0399	0.9937	0.9277	0.8243
		SD	0.0517	0.0554	0.0605	0.0700	0.0876	0.1514	0.1451	0.1476	0.1646	0.2088
RMSE		0.5043	0.3032	0.1297	0.1019	0.3378	0.1783	0.1504	0.1478	0.1797	0.2729	
	<i>Poly.</i>	<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		
MSII-FM-A	Mean	1.1180	1.1144	1.1147	1.1190		0.9565	0.9554	0.9554	0.9485		
	SD	0.1204	0.1208	0.1208	0.1221		0.1784	0.1804	0.1814	0.1847		
	RMSE	0.1685	0.1663	0.1666	0.1705		0.1837	0.1859	0.1868	0.1918		
	$\overline{SE}$	–	0.1241	0.1242	0.1249		–	0.1606	0.1608	0.1621		
	CR	–	89%	89%	89%		–	92%	91%	91%		
MSII-FM-B	Mean	1.1199	1.1145	1.1148	1.1191		0.9081	0.9529	0.9531	0.9461		
	SD	0.1207	0.1208	0.1209	0.1220		0.1789	0.1825	0.1833	0.1857		
	RMSE	0.1701	0.1664	0.1667	0.1705		0.2012	0.1885	0.1892	0.1934		
	$\overline{SE}$	–	0.1242	0.1242	0.1250		–	0.1603	0.1605	0.1616		
	CR	–	89%	89%	89%		–	91%	91%	90%		

**Table B2: Continued**

**Model B: Continued**

$(n, m)$	Estimator	$\beta_{22}$					$\gamma_1$					
(1000, 2000) $\Rightarrow \kappa = 1/2$	OLS*	Mean	0.9998					1.0010				
		SD	0.0201					0.0589				
		RMSE	0.0201					0.0589				
		$\overline{SE}$	0.0195					0.0587				
		CR	95%					96%				
		<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>
	MSOLS-A	Mean	0.5184	0.7075	0.8636	1.0004	1.1588	1.1253	1.0699	1.0277	0.9846	0.9264
		SD	0.0451	0.0482	0.0505	0.0534	0.0592	0.1357	0.1253	0.1239	0.1285	0.1409
		RMSE	0.4837	0.2965	0.1455	0.0534	0.1695	0.1848	0.1435	0.1270	0.1294	0.1589
	MSOLS-B	Mean	0.5188	0.7079	0.8640	1.0010	1.1598	1.1017	1.0518	1.0120	0.9687	0.9065
		SD	0.0452	0.0482	0.0506	0.0535	0.0593	0.1361	0.1257	0.1243	0.1296	0.1427
RMSE		0.4833	0.2961	0.1451	0.0535	0.1705	0.1699	0.1359	0.1249	0.1333	0.1706	
	<i>Poly.</i>	<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		
MSII-FM-A	Mean	1.0710	1.0700	1.0701	1.0693		0.9693	0.9687	0.9687	0.9662		
	SD	0.0981	0.0982	0.0981	0.0991		0.1530	0.1534	0.1537	0.1549		
	RMSE	0.1210	0.1206	0.1206	0.1209		0.1560	0.1565	0.1569	0.1586		
	$\overline{SE}$	–	0.0982	0.0982	0.0985		–	0.1472	0.1473	0.1479		
	CR	–	93%	93%	92%		–	95%	95%	94%		
MSII-FM-B	Mean	1.0718	1.0700	1.0701	1.0693		0.9455	0.9685	0.9683	0.9657		
	SD	0.0983	0.0982	0.0981	0.0991		0.1533	0.1545	0.1549	0.1555		
	RMSE	0.1217	0.1206	0.1206	0.1209		0.1627	0.1576	0.1581	0.1592		
	$\overline{SE}$	–	0.0982	0.0983	0.0985		–	0.1470	0.1470	0.1476		
	CR	–	93%	93%	92%		–	94%	94%	93%		
(2000, 1000) $\Rightarrow \kappa = 2$	OLS*	Mean	0.9995					1.0010				
		SD	0.0138					0.0396				
		RMSE	0.0138					0.0396				
		$\overline{SE}$	0.0139					0.0416				
		CR	95%					96%				
		<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>
	MSOLS-A	Mean	0.4978	0.7009	0.8843	1.0734	1.3253	1.1439	1.0811	1.0242	0.9602	0.8656
		SD	0.0401	0.0445	0.0499	0.0610	0.0794	0.1164	0.1139	0.1211	0.1422	0.1863
		RMSE	0.5038	0.3024	0.1260	0.0954	0.3348	0.1851	0.1398	0.1235	0.1476	0.2297
	MSOLS-B	Mean	0.4986	0.7018	0.8854	1.0748	1.3278	1.0979	1.0459	0.9932	0.9280	0.8242
		SD	0.0403	0.0446	0.0500	0.0612	0.0798	0.1167	0.1148	0.1231	0.1456	0.1937
RMSE		0.5030	0.3015	0.1251	0.0967	0.3373	0.1523	0.1236	0.1233	0.1624	0.2616	
	<i>Poly.</i>	<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		
MSII-FM-A	Mean	1.1109	1.1073	1.1077	1.1129		0.9613	0.9594	0.9594	0.9566		
	SD	0.1040	0.1045	0.1045	0.1043		0.1391	0.1400	0.1403	0.1439		
	RMSE	0.1520	0.1498	0.1501	0.1537		0.1444	0.1458	0.1461	0.1504		
	$\overline{SE}$	–	0.1030	0.1030	0.1036		–	0.1144	0.1146	0.1153		
	CR	–	77%	77%	74%		–	81%	81%	79%		
MSII-FM-B	Mean	1.1126	1.1073	1.1077	1.1128		0.9150	0.9591	0.9586	0.9561		
	SD	0.1042	0.1045	0.1046	0.1044		0.1394	0.1408	0.1415	0.1441		
	RMSE	0.1535	0.1498	0.1501	0.1537		0.1633	0.1466	0.1474	0.1506		
	$\overline{SE}$	–	0.1030	0.1031	0.1036		–	0.1142	0.1144	0.1150		
	CR	–	76%	76%	74%		–	80%	80%	80%		

Table B2: *Continued*

Model B: *Continued*

$(n, m)$	Estimator		$\beta_{22}$					$\gamma_1$				
(2000, 2000) $\Rightarrow \kappa = 1$	OLS*	<i>Mean</i>	0.9994					1.0011				
		<i>SD</i>	0.0141					0.0408				
		<i>RMSE</i>	0.0141					0.0408				
		$\overline{SE}$	0.0139					0.0416				
		<i>CR</i>	95%					95%				
		<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>
	MSOLS-A	<i>Mean</i>	0.5152	0.7058	0.8625	1.0002	1.1584	1.1267	1.0714	1.0237	0.9809	0.9242
		<i>SD</i>	0.0349	0.0370	0.0395	0.0429	0.0504	0.0971	0.0916	0.0909	0.0962	0.1082
		<i>RMSE</i>	0.4861	0.2966	0.1431	0.0429	0.1663	0.1596	0.1161	0.0939	0.0981	0.1321
	MSOLS-B	<i>Mean</i>	0.5155	0.7062	0.8629	1.0008	1.1593	1.1026	1.0532	1.0080	0.9652	0.9047
		<i>SD</i>	0.0350	0.0372	0.0396	0.0431	0.0506	0.0979	0.0921	0.0913	0.0973	0.1107
		<i>RMSE</i>	0.4857	0.2962	0.1427	0.0431	0.1672	0.1418	0.1063	0.0916	0.1033	0.1461
	<i>Poly.</i>	<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		
MSII-FM-A	<i>Mean</i>	1.0621	1.0610	1.0611	1.0610		0.9715	0.9707	0.9705	0.9682		
	<i>SD</i>	0.0747	0.0746	0.0746	0.0751		0.1097	0.1105	0.1105	0.1103		
	<i>RMSE</i>	0.0971	0.0963	0.0964	0.0968		0.1133	0.1143	0.1144	0.1147		
	$\overline{SE}$	–	0.0764	0.0764	0.0765		–	0.1045	0.1045	0.1051		
	<i>CR</i>	–	83%	82%	82%		–	87%	87%	87%		
MSII-FM-B	<i>Mean</i>	1.0629	1.0610	1.0611	1.0610		0.9471	0.9699	0.9698	0.9678		
	<i>SD</i>	0.0748	0.0746	0.0746	0.0752		0.1107	0.1119	0.1120	0.1119		
	<i>RMSE</i>	0.0978	0.0964	0.0964	0.0968		0.1227	0.1158	0.1160	0.1165		
	$\overline{SE}$	–	0.0764	0.0764	0.0766		–	0.1043	0.1044	0.1049		
	<i>CR</i>	–	82%	83%	82%		–	86%	86%	86%		
(2000, 4000) $\Rightarrow \kappa = 1/2$	OLS*	<i>Mean</i>	0.9999					1.0003				
		<i>SD</i>	0.0135					0.0400				
		<i>RMSE</i>	0.0135					0.0400				
		$\overline{SE}$	0.0139					0.0415				
		<i>CR</i>	96%					95%				
		<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>
	MSOLS-A	<i>Mean</i>	0.5263	0.7077	0.8497	0.9584	1.0627	1.1196	1.0705	1.0328	1.0023	0.9673
		<i>SD</i>	0.0317	0.0339	0.0354	0.0374	0.0395	0.0895	0.0857	0.0834	0.0835	0.0865
		<i>RMSE</i>	0.4748	0.2943	0.1544	0.0560	0.0741	0.1493	0.1110	0.0896	0.0835	0.0925
	MSOLS-B	<i>Mean</i>	0.5265	0.7079	0.8500	0.9587	1.0631	1.1078	1.0614	1.0250	0.9948	0.9585
		<i>SD</i>	0.0318	0.0340	0.0354	0.0374	0.0395	0.0898	0.0857	0.0834	0.0836	0.0870
		<i>RMSE</i>	0.4746	0.2941	0.1541	0.0557	0.0745	0.1403	0.1054	0.0870	0.0838	0.0963
	<i>Poly.</i>	<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		
MSII-FM-A	<i>Mean</i>	1.0385	1.0383	1.0383	1.0357		0.9841	0.9834	0.9835	0.9835		
	<i>SD</i>	0.0673	0.0674	0.0674	0.0677		0.0983	0.0984	0.0983	0.1013		
	<i>RMSE</i>	0.0776	0.0775	0.0775	0.0766		0.0996	0.0998	0.0997	0.1026		
	$\overline{SE}$	–	0.0649	0.0649	0.0649		–	0.0992	0.0992	0.0996		
	<i>CR</i>	–	83%	84%	85%		–	90%	90%	89%		
MSII-FM-B	<i>Mean</i>	1.0389	1.0383	1.0383	1.0357		0.9722	0.9833	0.9834	0.9834		
	<i>SD</i>	0.0674	0.0674	0.0674	0.0677		0.0988	0.0989	0.0988	0.1015		
	<i>RMSE</i>	0.0778	0.0775	0.0775	0.0766		0.1026	0.1003	0.1002	0.1028		
	$\overline{SE}$	–	0.0649	0.0649	0.0649		–	0.0991	0.0991	0.0994		
	<i>CR</i>	–	84%	84%	85%		–	90%	90%	89%		

**Table B2: Continued**

**Model C:**  $g_{22}(z) = 4\sqrt{|z/2|(1-|z/2|)}\sin\{2\pi(1+\epsilon)/(|z/2|+\epsilon)\}$ ,  $\epsilon = 0.05$

$(n, m)$	Estimator	$\beta_{22}$					$\gamma_1$					
(1000, 500) $\Rightarrow \kappa = 2$	OLS*	Mean	0.9998					0.9995				
		SD	0.0164					0.0598				
		RMSE	0.0164					0.0598				
		$\overline{SE}$	0.0164					0.0586				
		CR	95%					95%				
		<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>
	MSOLS-A	Mean	0.3955	0.5479	0.7069	0.8872	1.1755	1.0742	1.0368	0.9975	0.9445	0.8696
		SD	0.0608	0.0728	0.0967	0.1413	0.2329	0.2113	0.2130	0.2384	0.2908	0.3722
		RMSE	0.6076	0.4580	0.3086	0.1808	0.2916	0.2239	0.2161	0.2384	0.2960	0.3944
	MSOLS-B	Mean	0.3957	0.5482	0.7073	0.8881	1.1783	0.9863	0.9678	0.9355	0.8800	0.7880
SD		0.0610	0.0730	0.0972	0.1423	0.2348	0.2154	0.2150	0.2420	0.2989	0.3917	
RMSE		0.6074	0.4577	0.3085	0.1811	0.2948	0.2159	0.2174	0.2505	0.3221	0.4453	
	<i>Poly.</i>	<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		
MSII-FM-A	Mean	1.2714	1.2743	1.2751	1.2392		0.9455	0.9404	0.9413	0.9378		
	SD	0.2956	0.2968	0.2971	0.2967		0.2772	0.2838	0.2863	0.2841		
	RMSE	0.4013	0.4042	0.4049	0.3811		0.2825	0.2900	0.2923	0.2908		
	$\overline{SE}$	–	0.2996	0.2999	0.2966		–	0.2207	0.2215	0.2227		
	CR	–	97%	97%	97%		–	88%	88%	88%		
MSII-FM-B	Mean	1.2722	1.2742	1.2750	1.2390		0.8573	0.9375	0.9383	0.9359		
	SD	0.2960	0.2969	0.2972	0.2968		0.2803	0.2899	0.2936	0.2876		
	RMSE	0.4021	0.4041	0.4049	0.3810		0.3145	0.2965	0.3000	0.2947		
	$\overline{SE}$	–	0.2996	0.2999	0.2966		–	0.2206	0.2215	0.2221		
	CR	–	97%	97%	97%		–	87%	87%	87%		
(1000, 1000) $\Rightarrow \kappa = 1$	OLS*	Mean	0.9986					0.9977				
		SD	0.0165					0.0571				
		RMSE	0.0166					0.0572				
		$\overline{SE}$	0.0164					0.0588				
		CR	95%					95%				
		<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>
	MSOLS-A	Mean	0.4733	0.6337	0.7856	0.9459	1.1348	1.0597	1.0291	1.0100	0.9780	0.9319
		SD	0.0528	0.0571	0.0662	0.0847	0.1211	0.1767	0.1725	0.1766	0.1967	0.2391
		RMSE	0.5294	0.3707	0.2243	0.1005	0.1812	0.1865	0.1749	0.1769	0.1979	0.2486
	MSOLS-B	Mean	0.4735	0.6340	0.7858	0.9461	1.1355	1.0123	0.9931	0.9795	0.9457	0.8910
SD		0.0529	0.0573	0.0664	0.0850	0.1218	0.1782	0.1731	0.1786	0.1991	0.2466	
RMSE		0.5292	0.3705	0.2242	0.1006	0.1822	0.1786	0.1732	0.1798	0.2064	0.2696	
	<i>Poly.</i>	<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		
MSII-FM-A	Mean	1.1785	1.1803	1.1805	1.1588		0.9740	0.9723	0.9725	0.9667		
	SD	0.1768	0.1772	0.1773	0.1750		0.2100	0.2123	0.2133	0.2165		
	RMSE	0.2512	0.2528	0.2530	0.2363		0.2116	0.2141	0.2150	0.2191		
	$\overline{SE}$	–	0.1688	0.1689	0.1679		–	0.1869	0.1871	0.1891		
	CR	–	87%	87%	90%		–	92%	92%	92%		
MSII-FM-B	Mean	1.1791	1.1803	1.1805	1.1587		0.9272	0.9710	0.9714	0.9654		
	SD	0.1770	0.1772	0.1773	0.1750		0.2114	0.2153	0.2160	0.2185		
	RMSE	0.2518	0.2528	0.2530	0.2363		0.2236	0.2173	0.2179	0.2212		
	$\overline{SE}$	–	0.1688	0.1689	0.1679		–	0.1866	0.1868	0.1887		
	CR	–	87%	87%	90%		–	90%	90%	91%		

**Table B2: Continued**

**Model C: Continued**

$(n, m)$	Estimator		$\beta_{22}$					$\gamma_1$				
(1000, 2000) $\Rightarrow \kappa = 1/2$	OLS*	Mean	0.9996					1.0010				
		SD	0.0167					0.0589				
		RMSE	0.0167					0.0589				
		$\overline{SE}$	0.0164					0.0586				
		CR	95%					96%				
		<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>
	MSOLS-A	Mean	0.5383	0.6968	0.8378	0.9698	1.1154	1.0418	1.0200	1.0042	0.9840	0.9520
		SD	0.0456	0.0487	0.0538	0.0608	0.0789	0.1527	0.1419	0.1404	0.1503	0.1668
		RMSE	0.4639	0.3071	0.1709	0.0679	0.1398	0.1584	0.1433	0.1404	0.1511	0.1736
	MSOLS-B	Mean	0.5384	0.6968	0.8378	0.9699	1.1157	1.0186	1.0018	0.9885	0.9683	0.9326
SD		0.0457	0.0487	0.0538	0.0609	0.0791	0.1530	0.1424	0.1406	0.1519	0.1689	
RMSE		0.4639	0.3071	0.1709	0.0679	0.1401	0.1542	0.1424	0.1411	0.1552	0.1819	
	<i>Poly.</i>	<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		
MSII-FM-A	Mean	1.1289	1.1300	1.1302	1.1191		0.9807	0.9799	0.9799	0.9772		
	SD	0.1183	0.1176	0.1177	0.1183		0.1744	0.1748	0.1753	0.1770		
	RMSE	0.1749	0.1753	0.1755	0.1679		0.1755	0.1760	0.1765	0.1785		
	$\overline{SE}$	–	0.1141	0.1142	0.1140		–	0.1672	0.1673	0.1692		
	CR	–	83%	83%	86%		–	94%	94%	94%		
MSII-FM-B	Mean	1.1291	1.1300	1.1302	1.1191		0.9576	0.9802	0.9801	0.9774		
	SD	0.1186	0.1176	0.1177	0.1183		0.1752	0.1766	0.1771	0.1784		
	RMSE	0.1753	0.1753	0.1755	0.1679		0.1803	0.1777	0.1782	0.1799		
	$\overline{SE}$	–	0.1141	0.1142	0.1140		–	0.1670	0.1671	0.1689		
	CR	–	83%	83%	86%		–	94%	94%	94%		
(2000, 1000) $\Rightarrow \kappa = 2$	OLS*	Mean	0.9994					1.0009				
		SD	0.0117					0.0395				
		RMSE	0.0117					0.0396				
		$\overline{SE}$	0.0116					0.0415				
		CR	96%					96%				
		<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>
	MSOLS-A	Mean	0.4740	0.6327	0.7863	0.9445	1.1323	1.0610	1.0335	1.0078	0.9777	0.9314
		SD	0.0413	0.0459	0.0544	0.0743	0.1090	0.1360	0.1337	0.1446	0.1675	0.2109
		RMSE	0.5277	0.3701	0.2205	0.0928	0.1714	0.1491	0.1378	0.1448	0.1690	0.2218
	MSOLS-B	Mean	0.4741	0.6330	0.7865	0.9448	1.1328	1.0151	0.9984	0.9772	0.9458	0.8913
SD		0.0414	0.0461	0.0545	0.0746	0.1095	0.1363	0.1347	0.1465	0.1705	0.2180	
RMSE		0.5276	0.3699	0.2204	0.0928	0.1722	0.1372	0.1347	0.1482	0.1790	0.2436	
	<i>Poly.</i>	<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		
MSII-FM-A	Mean	1.1691	1.1709	1.1713	1.1526		0.9772	0.9743	0.9746	0.9734		
	SD	0.1419	0.1414	0.1415	0.1403		0.1699	0.1704	0.1705	0.1778		
	RMSE	0.2208	0.2219	0.2221	0.2073		0.1714	0.1723	0.1724	0.1798		
	$\overline{SE}$	–	0.1396	0.1396	0.1389		–	0.1321	0.1323	0.1337		
	CR	–	82%	82%	87%		–	87%	87%	85%		
MSII-FM-B	Mean	1.1694	1.1709	1.1713	1.1525		0.9313	0.9740	0.9738	0.9729		
	SD	0.1422	0.1414	0.1415	0.1404		0.1702	0.1715	0.1720	0.1782		
	RMSE	0.2212	0.2219	0.2221	0.2073		0.1836	0.1735	0.1739	0.1803		
	$\overline{SE}$	–	0.1396	0.1396	0.1389		–	0.1319	0.1321	0.1334		
	CR	–	82%	82%	86%		–	87%	86%	86%		

**Table B2: Continued**

**Model C: Continued**

$(n, m)$	Estimator		$\beta_{22}$					$\gamma_1$				
(2000, 2000) $\Rightarrow \kappa = 1$	OLS*	Mean	0.9997					1.0009				
		SD	0.0116					0.0405				
		RMSE	0.0116					0.0406				
		$\overline{SE}$	0.0116					0.0415				
		CR	95%					95%				
		<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>
	MSOLS-A	Mean	0.5365	0.6953	0.8374	0.9698	1.1123	1.0429	1.0200	1.0000	0.9811	0.9524
		SD	0.0350	0.0372	0.0421	0.0502	0.0646	0.1095	0.1049	0.1071	0.1160	0.1319
		RMSE	0.4648	0.3070	0.1680	0.0586	0.1295	0.1176	0.1068	0.1071	0.1175	0.1403
	MSOLS-B	Mean	0.5365	0.6953	0.8374	0.9699	1.1124	1.0192	1.0020	0.9844	0.9656	0.9333
		SD	0.0351	0.0372	0.0421	0.0503	0.0649	0.1105	0.1055	0.1077	0.1171	0.1347
		RMSE	0.4648	0.3069	0.1679	0.0587	0.1298	0.1121	0.1055	0.1089	0.1220	0.1503
(2000, 4000) $\Rightarrow \kappa = 1/2$	Poly.	<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		
		Mean	1.1229	1.1242	1.1243	1.1132	0.9787	0.9778	0.9776	0.9752		
		SD	0.0932	0.0933	0.0933	0.0924	0.1250	0.1254	0.1256	0.1274		
		RMSE	0.1543	0.1553	0.1554	0.1461	0.1268	0.1274	0.1276	0.1298		
		$\overline{SE}$	–	0.0894	0.0894	0.0892	–	0.1183	0.1183	0.1198		
		CR	–	63%	63%	69%	–	87%	87%	88%		
	MSII-FM-A	Mean	1.1230	1.1242	1.1243	1.1131	0.9548	0.9774	0.9772	0.9753		
		SD	0.0933	0.0933	0.0933	0.0924	0.1258	0.1267	0.1269	0.1288		
		RMSE	0.1544	0.1553	0.1554	0.1461	0.1337	0.1287	0.1289	0.1312		
	MSII-FM-B	$\overline{SE}$	–	0.0894	0.0894	0.0892	–	0.1182	0.1182	0.1196		
		CR	–	63%	63%	69%	–	86%	86%	87%		
(2000, 4000) $\Rightarrow \kappa = 1/2$	OLS*	Mean	1.0004					1.0003				
		SD	0.0115					0.0399				
		RMSE	0.0115					0.0399				
		$\overline{SE}$	0.0116					0.0414				
		CR	95%					95%				
		<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>
	MSOLS-A	Mean	0.5879	0.7403	0.8679	0.9784	1.0875	1.0367	1.0190	1.0052	0.9924	0.9741
		SD	0.0302	0.0327	0.0350	0.0393	0.0474	0.0992	0.0957	0.0944	0.0970	0.1036
		RMSE	0.4132	0.2618	0.1367	0.0448	0.0995	0.1057	0.0975	0.0945	0.0973	0.1068
	MSOLS-B	Mean	0.5879	0.7403	0.8679	0.9784	1.0876	1.0246	1.0095	0.9972	0.9848	0.9653
		SD	0.0302	0.0327	0.0351	0.0392	0.0474	0.0997	0.0957	0.0944	0.0973	0.1044
		RMSE	0.4132	0.2618	0.1367	0.0448	0.0996	0.1027	0.0962	0.0944	0.0985	0.1100
(2000, 4000) $\Rightarrow \kappa = 1/2$	Poly.	<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		
		Mean	1.0855	1.0861	1.0862	1.0803	0.9881	0.9872	0.9873	0.9869		
		SD	0.0655	0.0656	0.0656	0.0657	0.1101	0.1100	0.1099	0.1140		
		RMSE	0.1077	0.1083	0.1083	0.1038	0.1108	0.1107	0.1107	0.1148		
		$\overline{SE}$	–	0.0650	0.0650	0.0650	–	0.1094	0.1095	0.1106		
		CR	–	67%	67%	70%	–	90%	90%	90%		
	MSII-FM-A	Mean	1.0855	1.0861	1.0862	1.0803	0.9759	0.9868	0.9869	0.9864		
		SD	0.0656	0.0656	0.0656	0.0657	0.1107	0.1108	0.1107	0.1146		
		RMSE	0.1077	0.1083	0.1083	0.1038	0.1133	0.1116	0.1115	0.1154		
	MSII-FM-B	$\overline{SE}$	–	0.0650	0.0650	0.0650	–	0.1094	0.1094	0.1105		
		CR	–	67%	67%	70%	–	90%	90%	90%		

**Table B3:** Monte Carlo Results ( $d_3 = 3$ )

**Model A:**  $g_{22}(z) = z + (5/\tau)\phi(z/\tau)$ ,  $\tau = 0.75$

$(n, m)$	Estimator	$\beta_{22}$						$\gamma_1$						
(1000, 500) $\Rightarrow \kappa = 2$	OLS*	Mean	0.9998						1.0004					
		SD	0.0179						0.0595					
		RMSE	0.0179						0.0595					
		$\overline{SE}$	0.0182						0.0603					
		CR	95%						95%					
		<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>		
MSOLS-A	Mean	1.1216	0.9998	0.7247	0.3344	-0.1169	1.0739	1.0735	1.1935	1.3952	1.6127			
	SD	0.1232	0.1372	0.1787	0.2461	0.3599	0.4073	0.4165	0.4487	0.5282	0.6401			
	RMSE	0.1731	0.1372	0.3282	0.7096	1.1735	0.4139	0.4229	0.4887	0.6597	0.8860			
MSOLS-B	Mean	1.2502	1.1261	0.8567	0.4898	0.1097	-0.0781	0.1476	0.3845	0.6191	0.7544			
	SD	0.1258	0.1401	0.1806	0.2465	0.3741	0.4126	0.4306	0.4760	0.5700	0.7174			
	RMSE	0.2800	0.1885	0.2305	0.5667	0.9657	1.1544	0.9550	0.7780	0.6855	0.7582			
MSII-FM-A	Poly.	<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>				
	Mean	0.6853	1.1387	1.1345	0.9234		1.2147	0.8184	0.8224	0.9692				
	SD	0.3615	0.3842	0.3897	0.3940		0.5147	0.5294	0.5443	0.5404				
	RMSE	0.4793	0.4085	0.4122	0.4014		0.5577	0.5597	0.5726	0.5413				
	$\overline{SE}$	–	0.4538	0.4550	0.4675		–	0.4885	0.4927	0.4918				
MSII-FM-B	CR	–	96%	97%	99%		–	91%	90%	92%				
	Mean	0.9286	1.2006	1.1964	0.9696		-0.0206	0.4094	0.4122	0.5733				
	SD	0.3849	0.3774	0.3802	0.3898		0.5304	0.5803	0.5930	0.5700				
	RMSE	0.3914	0.4274	0.4280	0.3910		1.1502	0.8280	0.8349	0.7121				
	$\overline{SE}$	–	0.4966	0.4977	0.5114		–	0.5177	0.5215	0.5177				
MSII-FM-B	CR	–	97%	97%	99%		–	75%	75%	82%				
	<hr/>													
	(1000, 1000) $\Rightarrow \kappa = 1$	OLS*	Mean	1.0004						0.9992				
			SD	0.0187						0.0597				
			RMSE	0.0187						0.0597				
$\overline{SE}$			0.0182						0.0603					
CR			95%						95%					
<i>K</i>			<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>		
MSOLS-A	Mean	1.0105	0.9051	0.6807	0.3214	-0.1630	1.1301	1.1272	1.2316	1.4342	1.7089			
	SD	0.0962	0.1014	0.1199	0.1514	0.2128	0.3290	0.3183	0.3397	0.3873	0.4674			
	RMSE	0.0967	0.1388	0.3411	0.6953	1.1823	0.3538	0.3428	0.4112	0.5818	0.8491			
MSOLS-B	Mean	1.0902	0.9792	0.7550	0.4024	-0.0597	0.3008	0.4906	0.6911	0.9211	1.1527			
	SD	0.0979	0.1037	0.1216	0.1537	0.1537	0.3351	0.3263	0.3567	0.4138	0.4138			
	RMSE	0.1331	0.1057	0.2735	0.6170	1.0811	0.7754	0.6049	0.4719	0.4213	0.5276			
MSII-FM-A	Poly.	<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>				
	Mean	0.7877	1.0676	1.0640	0.9427		1.1580	0.9078	0.9081	0.9845				
	SD	0.2137	0.2263	0.2277	0.2261		0.3693	0.3679	0.3714	0.3544				
	RMSE	0.3012	0.2362	0.2366	0.2332		0.4017	0.3793	0.3826	0.3547				
	$\overline{SE}$	–	0.2679	0.2684	0.2682		–	0.3574	0.3591	0.3477				
MSII-FM-B	CR	–	97%	97%	98%		–	94%	94%	95%				
	Mean	0.9305	1.0881	1.0853	0.9586		0.2842	0.7065	0.7038	0.7863				
	SD	0.2231	0.2215	0.2224	0.2224		0.3831	0.4064	0.4118	0.3929				
	RMSE	0.2336	0.2384	0.2382	0.2262		0.8119	0.5013	0.5073	0.4472				
	$\overline{SE}$	–	0.2807	0.2811	0.2812		–	0.3668	0.3685	0.3547				
MSII-FM-B	CR	–	98%	98%	99%		–	85%	85%	89%				



**Table B3: Continued**

**Model A: Continued**

$(n, m)$	Estimator	$\beta_{22}$					$\gamma_1$					
(1000, 2000) $\Rightarrow \kappa = 1/2$	OLS*	Mean	1.0006					0.9983				
		SD	0.0185					0.0608				
		RMSE	0.0185					0.0608				
		$\overline{SE}$	0.0182					0.0602				
		CR	95%					95%				
		<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>
	MSOLS-A	Mean	0.9305	0.8640	0.7019	0.4255	0.0184	1.1432	1.1475	1.2235	1.3814	1.6197
		SD	0.0798	0.0808	0.0860	0.1045	0.1332	0.2679	0.2373	0.2348	0.2517	0.3003
		RMSE	0.1059	0.1582	0.3103	0.5839	0.9906	0.3038	0.2794	0.3241	0.4569	0.6886
	MSOLS-B	Mean	0.9816	0.9104	0.7459	0.4715	0.0713	0.5673	0.7149	0.8657	1.0485	1.2690
SD		0.0810	0.0826	0.0865	0.1047	0.1333	0.2793	0.2478	0.2420	0.2636	0.3155	
RMSE		0.0831	0.1218	0.2684	0.5388	0.9382	0.5150	0.3777	0.2768	0.2680	0.4146	
	<i>Poly.</i>	<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		
MSII-FM-A	Mean	0.8607	1.0333	1.0318	0.9702		1.1012	0.9424	0.9441	0.9881		
	SD	0.1523	0.1581	0.1587	0.1522		0.2958	0.2983	0.2991	0.2855		
	RMSE	0.2064	0.1616	0.1619	0.1551		0.3126	0.3039	0.3043	0.2858		
	$\overline{SE}$	–	0.1764	0.1766	0.1735		–	0.2864	0.2869	0.2755		
	CR	–	97%	97%	98%		–	95%	95%	94%		
MSII-FM-B	Mean	0.9509	1.0423	1.0406	0.9777		0.4943	0.8397	0.8414	0.8862		
	SD	0.1569	0.1563	0.1567	0.1509		0.3100	0.3239	0.3249	0.3139		
	RMSE	0.1644	0.1619	0.1618	0.1525		0.5931	0.3614	0.3615	0.3339		
	$\overline{SE}$	–	0.1807	0.1808	0.1778		–	0.2905	0.2911	0.2779		
	CR	–	97%	97%	98%		–	88%	88%	90%		
(2000, 1000) $\Rightarrow \kappa = 2$	OLS*	Mean	0.9996					1.0007				
		SD	0.0130					0.0429				
		RMSE	0.0130					0.0430				
		$\overline{SE}$	0.0129					0.0427				
		CR	95%					95%				
		<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>
	MSOLS-A	Mean	1.0144	0.9119	0.6848	0.3269	-0.1556	1.1267	1.1333	1.2447	1.4379	1.7020
		SD	0.0751	0.0828	0.1003	0.1365	0.1926	0.2567	0.2579	0.2797	0.3333	0.4144
		RMSE	0.0765	0.1209	0.3308	0.6868	1.1715	0.2862	0.2903	0.3717	0.5504	0.8152
	MSOLS-B	Mean	1.0956	0.9879	0.7600	0.4093	-0.0524	0.2957	0.4913	0.7009	0.9220	1.1476
SD		0.0775	0.0849	0.1022	0.1387	0.1967	0.2641	0.2731	0.2937	0.3518	0.4394	
RMSE		0.1230	0.0858	0.2609	0.6068	1.0706	0.7522	0.5774	0.4192	0.3604	0.4635	
	<i>Poly.</i>	<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		
MSII-FM-A	Mean	0.7951	1.0719	1.0690	0.9545		1.1584	0.9104	0.9139	0.9871		
	SD	0.1801	0.1857	0.1869	0.1819		0.2994	0.3037	0.3062	0.2898		
	RMSE	0.2728	0.1992	0.1993	0.1875		0.3387	0.3166	0.3180	0.2901		
	$\overline{SE}$	–	0.2412	0.2415	0.2434		–	0.2788	0.2798	0.2734		
	CR	–	98%	98%	99%		–	92%	91%	94%		
MSII-FM-B	Mean	0.9408	1.0946	1.0920	0.9718		0.2790	0.6999	0.7023	0.7832		
	SD	0.1895	0.1839	0.1845	0.1808		0.3125	0.3315	0.3355	0.3187		
	RMSE	0.1985	0.2069	0.2061	0.1830		0.7858	0.4471	0.4485	0.3854		
	$\overline{SE}$	–	0.2540	0.2543	0.2564		–	0.2887	0.2897	0.2817		
	CR	–	98%	98%	99%		–	77%	78%	85%		

**Table B3: Continued**

**Model A: Continued**

$(n, m)$	Estimator		$\beta_{22}$					$\gamma_1$				
(2000, 2000) $\Rightarrow \kappa = 1$	OLS*	Mean	0.9999					0.9992				
		SD	0.0131					0.0431				
		RMSE	0.0131					0.0431				
		$\overline{SE}$	0.0129					0.0427				
		CR	95%					94%				
		<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>
	MSOLS-A	Mean	0.9289	0.8668	0.7056	0.4301	0.0225	1.1461	1.1436	1.2127	1.3675	1.5993
		SD	0.0593	0.0631	0.0713	0.0862	0.1159	0.2027	0.1882	0.1866	0.2113	0.2614
		RMSE	0.0926	0.1474	0.3029	0.5764	0.9843	0.2499	0.2368	0.2829	0.4239	0.6539
	MSOLS-B	Mean	0.9793	0.9129	0.7491	0.4760	0.0770	0.5751	0.7141	0.8586	1.0349	1.2431
		SD	0.0609	0.0643	0.0722	0.0872	0.1162	0.2073	0.1934	0.1912	0.2192	0.2694
RMSE		0.0643	0.1082	0.2611	0.5312	0.9302	0.4728	0.3452	0.2378	0.2219	0.3629	
	<i>Poly.</i>	<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		
MSII-FM-A	Mean	0.8607	1.0296	1.0285	0.9666		1.1016	0.9464	0.9470	0.9884		
	SD	0.1172	0.1196	0.1200	0.1158		0.2236	0.2245	0.2252	0.2130		
	RMSE	0.1821	0.1232	0.1233	0.1205		0.2457	0.2308	0.2314	0.2133		
	$\overline{SE}$	–	0.1503	0.1504	0.1491		–	0.2124	0.2128	0.2053		
	CR	–	95%	96%	97%		–	88%	88%	88%		
MSII-FM-B	Mean	0.9492	1.0372	1.0360	0.9729		0.5015	0.8511	0.8517	0.8928		
	SD	0.1218	0.1189	0.1192	0.1153		0.2316	0.2453	0.2465	0.2304		
	RMSE	0.1319	0.1246	0.1245	0.1184		0.5497	0.2870	0.2877	0.2541		
	$\overline{SE}$	–	0.1544	0.1545	0.1533		–	0.2168	0.2172	0.2084		
	CR	–	96%	96%	98%		–	78%	79%	82%		
(2000, 4000) $\Rightarrow \kappa = 1/2$	OLS*	Mean	1.0006					0.9997				
		SD	0.0125					0.0426				
		RMSE	0.0125					0.0426				
		$\overline{SE}$	0.0129					0.0427				
		CR	96%					96%				
		<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>
	MSOLS-A	Mean	0.8680	0.8456	0.7484	0.5620	0.2663	1.1698	1.1454	1.1866	1.2876	1.4625
		SD	0.0476	0.0493	0.0522	0.0598	0.0740	0.1609	0.1463	0.1462	0.1576	0.1774
		RMSE	0.1403	0.1621	0.2569	0.4421	0.7374	0.2339	0.2063	0.2371	0.3279	0.4954
	MSOLS-B	Mean	0.8998	0.8740	0.7747	0.5892	0.2976	0.7801	0.8600	0.9558	1.0731	1.2347
		SD	0.0483	0.0499	0.0530	0.0601	0.0747	0.1654	0.1525	0.1513	0.1637	0.1870
RMSE		0.1112	0.1356	0.2315	0.4152	0.7063	0.2751	0.2071	0.1576	0.1793	0.3001	
	<i>Poly.</i>	<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		
MSII-FM-A	Mean	0.9108	1.0158	1.0154	0.9846		1.0662	0.9697	0.9704	0.9822		
	SD	0.0849	0.0862	0.0866	0.0841		0.1728	0.1705	0.1713	0.1708		
	RMSE	0.1231	0.0876	0.0879	0.0855		0.1850	0.1732	0.1738	0.1717		
	$\overline{SE}$	–	0.1021	0.1021	0.1003		–	0.1734	0.1735	0.1673		
	CR	–	98%	98%	98%		–	95%	95%	94%		
MSII-FM-B	Mean	0.9661	1.0188	1.0184	0.9869		0.6581	0.9241	0.9252	0.9390		
	SD	0.0871	0.0855	0.0855	0.0831		0.1783	0.1827	0.1835	0.1801		
	RMSE	0.0935	0.0875	0.0874	0.0841		0.3856	0.1978	0.1982	0.1902		
	$\overline{SE}$	–	0.1036	0.1036	0.1018		–	0.1752	0.1754	0.1685		
	CR	–	98%	98%	98%		–	92%	92%	92%		

**Table B3: Continued**

**Model B:**  $g_{22}(z) = 2|z|$

$(n, m)$	Estimator	$\beta_{22}$						$\gamma_1$					
(1000, 500) $\Rightarrow \kappa = 2$	OLS*	Mean	1.0003						1.0001				
		SD	0.0161						0.0581				
		RMSE	0.0161						0.0581				
		$\overline{SE}$	0.0161						0.0586				
		CR	95%						96%				
		<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	
	MSOLS-A	Mean	0.2363	0.5494	0.8987	1.2655	1.6227	1.3068	1.1610	1.0081	0.8499	0.6838	
		SD	0.0821	0.0945	0.1140	0.1561	0.2216	0.3164	0.3359	0.3733	0.4421	0.5264	
		RMSE	0.7681	0.4604	0.1525	0.3080	0.6610	0.4407	0.3725	0.3734	0.4669	0.6141	
	MSOLS-B	Mean	0.2465	0.5623	0.9171	1.2958	1.6881	0.6130	0.6130	0.5331	0.3937	0.1744	
		SD	0.0835	0.0952	0.1154	0.1580	0.2267	0.3285	0.3521	0.3992	0.4879	0.6110	
RMSE		0.7581	0.4479	0.1421	0.3353	0.7244	0.5076	0.5231	0.6143	0.7782	1.0271		
	<i>Poly.</i>	<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>			
MSII-FM-A	Mean	1.1273	1.0590	1.0632	1.2896		0.9368	0.9271	0.9273	0.8593			
	SD	0.2386	0.2417	0.2426	0.2779		0.4106	0.4239	0.4339	0.4503			
	RMSE	0.2704	0.2487	0.2506	0.4014		0.4154	0.4301	0.4400	0.4717			
	$\overline{SE}$	–	0.2377	0.2386	0.2546		–	0.3281	0.3323	0.3396			
	CR	–	96%	96%	86%		–	88%	87%	84%			
MSII-FM-B	Mean	1.1585	1.0592	1.0636	1.2937		0.2298	0.6899	0.6893	0.6180			
	SD	0.2436	0.2419	0.2428	0.2796		0.4257	0.4833	0.4963	0.4900			
	RMSE	0.2907	0.2490	0.2510	0.4056		0.8800	0.5742	0.5855	0.6213			
	$\overline{SE}$	–	0.2399	0.2407	0.2574		–	0.3270	0.3311	0.3340			
	CR	–	96%	96%	86%		–	73%	73%	71%			
(1000, 1000) $\Rightarrow \kappa = 1$	OLS*	Mean	0.9995						0.9997				
		SD	0.0162						0.0580				
		RMSE	0.0162						0.0580				
		$\overline{SE}$	0.0161						0.0586				
		CR	95%						96%				
		<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	
	MSOLS-A	Mean	0.3731	0.6713	0.9796	1.3117	1.6686	1.2515	1.1158	0.9867	0.8516	0.7005	
		SD	0.0704	0.0737	0.0829	0.1037	0.1391	0.2681	0.2678	0.2918	0.3287	0.3905	
		RMSE	0.6309	0.3368	0.0854	0.3285	0.6829	0.3676	0.2918	0.2921	0.3607	0.4921	
	MSOLS-B	Mean	0.3803	0.6796	0.9898	1.3268	1.6963	0.7441	0.7329	0.6641	0.5418	0.3553	
		SD	0.0715	0.0743	0.0840	0.1050	0.1417	0.2729	0.2737	0.3061	0.3520	0.4288	
RMSE		0.6238	0.3289	0.0846	0.3432	0.7106	0.3741	0.3824	0.4545	0.5778	0.7743		
	<i>Poly.</i>	<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>			
MSII-FM-A	Mean	1.0871	1.0445	1.0461	1.1812		0.9672	0.9593	0.9569	0.9118			
	SD	0.1532	0.1539	0.1543	0.1620		0.3164	0.3197	0.3215	0.3241			
	RMSE	0.1762	0.1602	0.1610	0.2431		0.3180	0.3223	0.3243	0.3359			
	$\overline{SE}$	–	0.1515	0.1518	0.1569		–	0.2750	0.2764	0.2759			
	CR	–	89%	89%	71%		–	82%	83%	82%			
MSII-FM-B	Mean	1.1042	1.0443	1.0461	1.1823		0.4556	0.8377	0.8331	0.7883			
	SD	0.1553	0.1539	0.1542	0.1619		0.3247	0.3595	0.3636	0.3618			
	RMSE	0.1871	0.1601	0.1609	0.2438		0.6339	0.3944	0.4001	0.4192			
	$\overline{SE}$	–	0.1521	0.1524	0.1576		–	0.2758	0.2774	0.2746			
	CR	–	89%	90%	71%		–	76%	75%	73%			

**Table B3: Continued**

**Model B: Continued**

$(n, m)$	Estimator		$\beta_{22}$					$\gamma_1$				
(1000, 2000) $\Rightarrow \kappa = 1/2$	OLS*	Mean	0.9994					0.9988				
		SD	0.0164					0.0591				
		RMSE	0.0164					0.0591				
		$\overline{SE}$	0.0161					0.0585				
		CR	94%					94%				
		K	1	2	4	8	16	1	2	4	8	16
	MSOLS-A	Mean	0.4706	0.7356	0.9960	1.2694	1.5771	1.1907	1.0924	0.9896	0.8783	0.7489
		SD	0.0565	0.0581	0.0623	0.0724	0.0903	0.2219	0.2118	0.2103	0.2271	0.2642
		RMSE	0.5324	0.2707	0.0624	0.2790	0.5841	0.2926	0.2311	0.2106	0.2577	0.3645
	MSOLS-B	Mean	0.4758	0.7407	1.0018	1.2769	1.5890	0.8343	0.8300	0.7738	0.6767	0.5315
		SD	0.0564	0.0582	0.0624	0.0729	0.0908	0.2279	0.2151	0.2153	0.2366	0.2781
		RMSE	0.5272	0.2657	0.0624	0.2863	0.5959	0.2817	0.2741	0.3123	0.4006	0.5448
MSII-FM-A	Poly.	(initial)	2nd	3rd	4th	(initial)	2nd	3rd	4th			
	Mean	1.0590	1.0328	1.0338	1.1133	0.9730	0.9675	0.9677	0.9456			
	SD	0.1085	0.1103	0.1105	0.1136	0.2456	0.2499	0.2507	0.2487			
	RMSE	0.1236	0.1150	0.1155	0.1604	0.2471	0.2520	0.2527	0.2546			
	$\overline{SE}$	–	0.1077	0.1078	0.1097	–	0.2358	0.2362	0.2344			
	CR	–	94%	94%	85%	–	94%	94%	93%			
MSII-FM-B	Mean	1.0694	1.0330	1.0340	1.1139	0.6140	0.9033	0.9035	0.8809			
	SD	0.1090	0.1099	0.1101	0.1132	0.2514	0.2742	0.2754	0.2743			
	RMSE	0.1292	0.1147	0.1153	0.1606	0.4606	0.2908	0.2919	0.2990			
	$\overline{SE}$	–	0.1079	0.1080	0.1099	–	0.2368	0.2373	0.2341			
	CR	–	94%	94%	85%	–	90%	88%	87%			
	(2000, 1000) $\Rightarrow \kappa = 2$	OLS*	Mean	0.9999					1.0004			
SD			0.0117					0.0416				
RMSE			0.0117					0.0416				
$\overline{SE}$			0.0114					0.0415				
CR			94%					95%				
K			1	2	4	8	16	1	2	4	8	16
MSOLS-A		Mean	0.3704	0.6684	0.9765	1.3096	1.6622	1.2498	1.1254	1.0016	0.8592	0.7033
		SD	0.0519	0.0601	0.0707	0.0902	0.1214	0.2071	0.2168	0.2406	0.2867	0.3501
		RMSE	0.6318	0.3370	0.0745	0.3224	0.6732	0.3244	0.2505	0.2406	0.3194	0.4589
MSOLS-B		Mean	0.3773	0.6762	0.9863	1.3241	1.6890	0.7421	0.7401	0.6780	0.5496	0.3627
		SD	0.0524	0.0605	0.0715	0.0914	0.1237	0.2126	0.2277	0.2517	0.3050	0.3811
		RMSE	0.6249	0.3294	0.0728	0.3368	0.7000	0.3342	0.3456	0.4087	0.5440	0.7426
MSII-FM-A	Poly.	(initial)	2nd	3rd	4th	(initial)	2nd	3rd	4th			
	Mean	1.0791	1.0366	1.0385	1.1737	0.9747	0.9672	0.9678	0.9240			
	SD	0.1247	0.1256	0.1258	0.1310	0.2475	0.2546	0.2563	0.2530			
	RMSE	0.1476	0.1308	0.1316	0.2176	0.2488	0.2567	0.2583	0.2641			
	$\overline{SE}$	–	0.1276	0.1341	0.1327	–	0.1956	0.2041	0.1962			
	CR	–	95%	94%	79%	–	87%	86%	86%			
MSII-FM-B	Mean	1.0959	1.0367	1.0386	1.1750	0.4599	0.8387	0.8391	0.7963			
	SD	0.1260	0.1252	0.1252	0.1307	0.2544	0.2814	0.2849	0.2802			
	RMSE	0.1583	0.1305	0.1311	0.2184	0.5970	0.3244	0.3272	0.3464			
	$\overline{SE}$	–	0.1281	0.1283	0.1334	–	0.1965	0.1976	0.1953			
	CR	–	95%	95%	80%	–	77%	77%	73%			

**Table B3: Continued**

**Model B: Continued**

$(n, m)$	Estimator	$\beta_{22}$					$\gamma_1$					
(2000, 2000) $\implies \kappa = 1$	OLS*	Mean	1.0005					0.9990				
		SD	0.0108					0.0420				
		RMSE	0.0109					0.0420				
		$\overline{SE}$	0.0114					0.0416				
		CR	97%					94%				
		K	1	2	4	8	16	1	2	4	8	16
	MSOLS-A	Mean	0.4721	0.7349	0.9930	1.2668	1.5746	1.1927	1.0914	0.9839	0.8710	0.7361
		SD	0.0402	0.0437	0.0503	0.0609	0.0786	0.1712	0.1625	0.1671	0.1896	0.2302
		RMSE	0.5295	0.2687	0.0508	0.2737	0.5799	0.2578	0.1864	0.1679	0.2293	0.3502
	MSOLS-B	Mean	0.4772	0.7401	0.9988	1.2744	1.5869	0.8413	0.8312	0.7712	0.6700	0.5149
		SD	0.0406	0.0440	0.0504	0.0611	0.0791	0.1717	0.1646	0.1699	0.1961	0.2395
		RMSE	0.5244	0.2636	0.0504	0.2812	0.5922	0.2338	0.2358	0.2850	0.3839	0.5410
MSII-FM-A	Poly.	(initial)	2nd	3rd	4th		(initial)	2nd	3rd	4th		
	Mean	1.0596	1.0337	1.0344	1.1126		0.9735	0.9678	0.9675	0.9431		
	SD	0.0819	0.0827	0.0828	0.0865		0.1888	0.1907	0.1908	0.1892		
	RMSE	0.1013	0.0892	0.0897	0.1420		0.1907	0.1934	0.1936	0.1975		
	$\overline{SE}$	–	0.0858	0.0858	0.0875		–	0.1670	0.1674	0.1658		
	CR	–	95%	95%	77%		–	90%	91%	90%		
MSII-FM-B	Mean	1.0699	1.0337	1.0344	1.1131		0.6198	0.9096	0.9093	0.8834		
	SD	0.0828	0.0826	0.0828	0.0865		0.1929	0.2087	0.2095	0.2047		
	RMSE	0.1084	0.0892	0.0897	0.1424		0.4263	0.2274	0.2283	0.2356		
	$\overline{SE}$	–	0.0859	0.0860	0.0877		–	0.1685	0.1689	0.1661		
	CR	–	95%	95%	77%		–	86%	86%	83%		
	(2000, 4000) $\implies \kappa = 1/2$	OLS*	Mean	0.9996					1.0002			
SD			0.0117					0.0414				
RMSE			0.0117					0.0414				
$\overline{SE}$			0.0114					0.0415				
CR			95%					95%				
K			1	2	4	8	16	1	2	4	8	16
MSOLS-A		Mean	0.5408	0.7702	0.9797	1.1909	1.4316	1.1577	1.0708	0.9919	0.9042	0.8036
		SD	0.0356	0.0369	0.0392	0.0446	0.0545	0.1374	0.1271	0.1294	0.1416	0.1586
		RMSE	0.4606	0.2328	0.0442	0.1960	0.4350	0.2092	0.1455	0.1297	0.1710	0.2525
MSOLS-B		Mean	0.5441	0.7735	0.9831	1.1950	1.4374	0.9161	0.8968	0.8526	0.7746	0.6635
		SD	0.0358	0.0371	0.0395	0.0449	0.0548	0.1412	0.1315	0.1331	0.1473	0.1667
		RMSE	0.4573	0.2295	0.0429	0.2001	0.4408	0.1642	0.1671	0.1986	0.2692	0.3755
MSII-FM-A	Poly.	(initial)	2nd	3rd	4th		(initial)	2nd	3rd	4th		
	Mean	1.0409	1.0252	1.0255	1.0696		0.9828	0.9793	0.9796	0.9595		
	SD	0.0610	0.0614	0.0614	0.0632		0.1481	0.1483	0.1487	0.1524		
	RMSE	0.0735	0.0664	0.0665	0.0940		0.1491	0.1497	0.1501	0.1577		
	$\overline{SE}$	–	0.0641	0.0641	0.0647		–	0.1457	0.1459	0.1445		
	CR	–	95%	95%	82%		–	95%	94%	92%		
MSII-FM-B	Mean	1.0469	1.0252	1.0255	1.0697		0.7398	0.9511	0.9517	0.9324		
	SD	0.0614	0.0613	0.0613	0.0631		0.1512	0.1594	0.1604	0.1620		
	RMSE	0.0772	0.0663	0.0664	0.0940		0.3009	0.1667	0.1675	0.1755		
	$\overline{SE}$	–	0.0641	0.0642	0.0648		–	0.1464	0.1466	0.1447		
	CR	–	95%	95%	83%		–	91%	91%	90%		

**Table B3: Continued**

**Model C:**  $g_{22}(z) = 4\sqrt{|z/2|(1 - |z/2|)} \sin\{2\pi(1 + \epsilon) / (|z/2| + \epsilon)\}$ ,  $\epsilon = 0.05$

$(n, m)$	Estimator	$\beta_{22}$					$\gamma_1$					
(1000, 500) $\Rightarrow \kappa = 2$	OLS*	Mean	1.0000					1.0004				
		SD	0.0141					0.0580				
		RMSE	0.0141					0.0580				
		$\overline{SE}$	0.0139					0.0585				
		CR	94%					95%				
		<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>
	MSOLS-A	Mean	0.1411	0.2718	0.4936	0.9182	1.7621	1.1797	1.0863	0.9996	0.9106	0.7789
		SD	0.0924	0.1171	0.1571	0.2391	0.4217	0.3710	0.3812	0.4103	0.4737	0.5816
		RMSE	0.8638	0.7376	0.5302	0.2527	0.8710	0.4123	0.3908	0.4103	0.4821	0.6223
	MSOLS-B	Mean	0.1424	0.2741	0.4978	0.9293	1.7980	0.4887	0.5422	0.5281	0.4619	0.2917
SD		0.0932	0.1176	0.1578	0.2403	0.4264	0.3778	0.3963	0.4331	0.5164	0.6687	
RMSE		0.8627	0.7354	0.5264	0.2505	0.9048	0.6357	0.6055	0.6405	0.7458	0.9741	
	<i>Poly.</i>	<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		
MSII-FM-A	Mean	1.3813	1.3024	1.3108	1.3214		0.9379	0.9091	0.9112	0.8966		
	SD	1.0069	1.0051	1.0179	1.1190		0.5240	0.5362	0.5495	0.5593		
	RMSE	1.0767	1.0496	1.0642	1.1643		0.5277	0.5439	0.5566	0.5688		
	$\overline{SE}$	–	0.8795	0.8859	0.9109		–	0.4318	0.4369	0.4397		
	CR	–	94%	93%	94%		–	82%	81%	81%		
MSII-FM-B	Mean	1.3958	1.3013	1.3091	1.3184		0.2421	0.6741	0.6740	0.6597		
	SD	1.0155	1.0025	1.0128	1.1083		0.5232	0.5876	0.6031	0.5840		
	RMSE	1.0899	1.0468	1.0589	1.1531		0.9210	0.6719	0.6856	0.6759		
	$\overline{SE}$	–	0.8807	0.8865	0.9108		–	0.4252	0.4300	0.4265		
	CR	–	94%	93%	95%		–	70%	69%	69%		
(1000, 1000) $\Rightarrow \kappa = 1$	OLS*	Mean	0.9994					0.9997				
		SD	0.0135					0.0580				
		RMSE	0.0135					0.0580				
		$\overline{SE}$	0.0139					0.0585				
		CR	96%					96%				
		<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>
	MSOLS-A	Mean	0.2193	0.3687	0.5758	0.8942	1.5035	1.1498	1.0658	0.9978	0.9333	0.8457
		SD	0.0748	0.0887	0.1163	0.1528	0.2340	0.3103	0.3050	0.3246	0.3601	0.4262
		RMSE	0.7843	0.6375	0.4398	0.1859	0.5552	0.3445	0.3121	0.3246	0.3663	0.4532
	MSOLS-B	Mean	0.2205	0.3703	0.5788	0.8994	1.5166	0.6439	0.6835	0.6775	0.6299	0.5144
SD		0.0755	0.0895	0.1168	0.1542	0.2370	0.3176	0.3146	0.3406	0.3837	0.4664	
RMSE		0.7832	0.6360	0.4371	0.1842	0.5684	0.4772	0.4463	0.4691	0.5332	0.6733	
	<i>Poly.</i>	<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		
MSII-FM-A	Mean	1.1151	1.0889	1.0901	1.0651		0.9763	0.9550	0.9534	0.9404		
	SD	0.4064	0.4009	0.4005	0.3953		0.3698	0.3751	0.3770	0.3712		
	RMSE	0.4224	0.4106	0.4105	0.4007		0.3705	0.3778	0.3799	0.3760		
	$\overline{SE}$	–	0.3718	0.3726	0.3669		–	0.3288	0.3304	0.3245		
	CR	–	92%	92%	91%		–	85%	85%	86%		
MSII-FM-B	Mean	1.1217	1.0890	1.0903	1.0649		0.4709	0.8358	0.8318	0.8200		
	SD	0.4099	0.4012	0.4009	0.3954		0.3777	0.4210	0.4249	0.4136		
	RMSE	0.4275	0.4110	0.4110	0.4007		0.6501	0.4519	0.4570	0.4511		
	$\overline{SE}$	–	0.3722	0.3730	0.3673		–	0.3273	0.3290	0.3200		
	CR	–	92%	92%	91%		–	77%	77%	76%		

**Table B3: Continued**

**Model C: Continued**

$(n, m)$	Estimator		$\beta_{22}$					$\gamma_1$				
(1000, 2000) $\Rightarrow \kappa = 1/2$	OLS*	Mean	1.0007					0.9987				
		SD	0.0139					0.0589				
		RMSE	0.0139					0.0589				
		$\overline{SE}$	0.0139					0.0584				
		CR	95%					95%				
		<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>
	MSOLS-A	Mean	0.2986	0.4622	0.6628	0.9166	1.2987	1.1071	1.0556	1.0025	0.9469	0.8785
		SD	0.0611	0.0689	0.0858	0.1115	0.1546	0.2610	0.2493	0.2474	0.2608	0.2978
		RMSE	0.7040	0.5422	0.3480	0.1393	0.3363	0.2821	0.2554	0.2475	0.2661	0.3217
	MSOLS-B	Mean	0.2995	0.4634	0.6643	0.9190	1.3038	0.7526	0.7953	0.7895	0.7492	0.6670
		SD	0.0617	0.0693	0.0862	0.1121	0.1555	0.2659	0.2499	0.2505	0.2688	0.3134
		RMSE	0.7032	0.5411	0.3466	0.1383	0.3413	0.3632	0.3230	0.3272	0.3677	0.4573
	<i>Poly.</i>	<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		
MSII-FM-A	Mean	1.0628	1.0538	1.0548	1.0260		0.9831	0.9690	0.9694	0.9679		
	SD	0.2291	0.2295	0.2303	0.2254		0.2985	0.3018	0.3029	0.2987		
	RMSE	0.2376	0.2357	0.2367	0.2268		0.2990	0.3034	0.3044	0.3004		
	$\overline{SE}$	–	0.2326	0.2328	0.2287		–	0.2821	0.2826	0.2789		
	CR	–	92%	92%	91%		–	88%	88%	87%		
MSII-FM-B	Mean	1.0660	1.0538	1.0548	1.0258		0.6275	0.9051	0.9057	0.9038		
	SD	0.2308	0.2294	0.2302	0.2253		0.3033	0.3302	0.3315	0.3262		
	RMSE	0.2401	0.2356	0.2366	0.2267		0.4803	0.3436	0.3446	0.3401		
	$\overline{SE}$	–	0.2327	0.2329	0.2288		–	0.2823	0.2830	0.2771		
	CR	–	92%	92%	91%		–	81%	82%	82%		
(2000, 1000) $\Rightarrow \kappa = 2$	OLS*	Mean	1.0003					1.0004				
		SD	0.0101					0.0416				
		RMSE	0.0101					0.0416				
		$\overline{SE}$	0.0098					0.0414				
		CR	94%					95%				
		<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>
	MSOLS-A	Mean	0.2211	0.3747	0.5831	0.9049	1.5066	1.1452	1.0718	1.0079	0.9321	0.8381
		SD	0.0553	0.0683	0.0920	0.1298	0.2091	0.2451	0.2486	0.2668	0.3085	0.3753
		RMSE	0.7809	0.6290	0.4270	0.1609	0.5480	0.2849	0.2587	0.2669	0.3159	0.4087
	MSOLS-B	Mean	0.2220	0.3760	0.5853	0.9098	1.5189	0.6377	0.6881	0.6873	0.6292	0.5113
		SD	0.0556	0.0687	0.0931	0.1313	0.2118	0.2469	0.2579	0.2755	0.3275	0.4091
		RMSE	0.7800	0.6278	0.4250	0.1593	0.5605	0.4384	0.4048	0.4167	0.4947	0.6374
	<i>Poly.</i>	<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		
MSII-FM-A	Mean	1.1149	1.0863	1.0877	1.0644		0.9773	0.9577	0.9593	0.9470		
	SD	0.3100	0.3091	0.3101	0.3007		0.2945	0.3046	0.3075	0.2941		
	RMSE	0.3306	0.3209	0.3222	0.3075		0.2953	0.3075	0.3102	0.2988		
	$\overline{SE}$	–	0.3010	0.3016	0.2983		–	0.2318	0.2329	0.2289		
	CR	–	92%	92%	92%		–	79%	78%	78%		
MSII-FM-B	Mean	1.1202	1.0860	1.0873	1.0638		0.4662	0.8278	0.8287	0.8188		
	SD	0.3119	0.3092	0.3101	0.3007		0.2932	0.3264	0.3308	0.3158		
	RMSE	0.3343	0.3209	0.3222	0.3074		0.6090	0.3691	0.3725	0.3641		
	$\overline{SE}$	–	0.3013	0.3018	0.2986		–	0.2314	0.2326	0.2259		
	CR	–	92%	92%	92%		–	70%	69%	69%		

**Table B3: Continued**

Model C: Continued

$(n, m)$	Estimator	$\beta_{22}$					$\gamma_1$					
(2000, 2000) $\Rightarrow \kappa = 1$	OLS*	Mean	1.0002					0.9991				
		SD	0.0096					0.0419				
		RMSE	0.0096					0.0419				
		$\overline{SE}$	0.0099					0.0415				
		CR	96%					94%				
		<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>
	MSOLS-A	Mean	0.2994	0.4653	0.6632	0.9149	1.2939	1.1037	1.0492	0.9910	0.9347	0.8633
		SD	0.0454	0.0541	0.0657	0.0877	0.1231	0.2007	0.1904	0.1946	0.2169	0.2535
		RMSE	0.7021	0.5374	0.3432	0.1222	0.3187	0.2259	0.1967	0.1948	0.2265	0.2880
	MSOLS-B	Mean	0.3003	0.4664	0.6648	0.9175	1.2991	0.7534	0.7911	0.7804	0.7366	0.6483
		SD	0.0459	0.0546	0.0664	0.0886	0.1243	0.2033	0.1931	0.1977	0.2235	0.2622
RMSE		0.7012	0.5364	0.3417	0.1211	0.3239	0.3195	0.2845	0.2955	0.3454	0.4387	
	<i>Poly.</i>	<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		
MSII-FM-A	Mean	1.0576	1.0477	1.0481	1.0191		0.9800	0.9667	0.9663	0.9617		
	SD	0.1826	0.1816	0.1818	0.1787		0.2277	0.2305	0.2307	0.2239		
	RMSE	0.1915	0.1877	0.1881	0.1797		0.2286	0.2328	0.2332	0.2271		
	$\overline{SE}$	–	0.1800	0.1802	0.1771		–	0.1985	0.1989	0.1961		
	CR	–	97%	97%	96%		–	90%	90%	91%		
MSII-FM-B	Mean	1.0608	1.0477	1.0481	1.0189		0.6300	0.9094	0.9094	0.9032		
	SD	0.1840	0.1817	0.1819	0.1788		0.2328	0.2526	0.2530	0.2438		
	RMSE	0.1938	0.1879	0.1882	0.1798		0.4371	0.2683	0.2687	0.2623		
	$\overline{SE}$	–	0.1801	0.1802	0.1772		–	0.1996	0.2000	0.1955		
	CR	–	96%	96%	96%		–	84%	85%	85%		
(2000, 4000) $\Rightarrow \kappa = 1/2$	OLS*	Mean	0.9996					1.0002				
		SD	0.0101					0.0413				
		RMSE	0.0101					0.0413				
		$\overline{SE}$	0.0099					0.0414				
		CR	94%					96%				
		<i>K</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>	<i>1</i>	<i>2</i>	<i>4</i>	<i>8</i>	<i>16</i>
	MSOLS-A	Mean	0.3710	0.5431	0.7357	0.9459	1.2056	1.0850	1.0386	0.9990	0.9525	0.9005
		SD	0.0392	0.0442	0.0498	0.0609	0.0823	0.1632	0.1510	0.1552	0.1676	0.1832
		RMSE	0.6302	0.4590	0.2690	0.0814	0.2215	0.1840	0.1559	0.1552	0.1742	0.2084
	MSOLS-B	Mean	0.3715	0.5439	0.7366	0.9472	1.2078	0.8424	0.8644	0.8599	0.8239	0.7632
		SD	0.0393	0.0444	0.0500	0.0610	0.0826	0.1675	0.1556	0.1586	0.1736	0.1910
RMSE		0.6297	0.4583	0.2681	0.0807	0.2236	0.2300	0.2064	0.2116	0.2473	0.3042	
	<i>Poly.</i>	<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		<i>(initial)</i>	<i>2nd</i>	<i>3rd</i>	<i>4th</i>		
MSII-FM-A	Mean	1.0282	1.0256	1.0257	0.9994		0.9895	0.9816	0.9820	0.9723		
	SD	0.1203	0.1214	0.1214	0.1187		0.1809	0.1807	0.1811	0.1849		
	RMSE	0.1235	0.1241	0.1241	0.1187		0.1812	0.1817	0.1820	0.1870		
	$\overline{SE}$	–	0.1206	0.1206	0.1188		–	0.1742	0.1744	0.1730		
	CR	–	95%	95%	96%		–	94%	94%	93%		
MSII-FM-B	Mean	1.0298	1.0257	1.0258	0.9994		0.7466	0.9515	0.9522	0.9437		
	SD	0.1203	0.1213	0.1213	0.1186		0.1849	0.1959	0.1964	0.1983		
	RMSE	0.1240	0.1240	0.1240	0.1186		0.3137	0.2018	0.2021	0.2061		
	$\overline{SE}$	–	0.1206	0.1206	0.1188		–	0.1749	0.1751	0.1729		
	CR	–	95%	95%	96%		–	91%	91%	91%		



## C An Order Selection Rule for the Polynomial Approximation in MSII-FM

An important practical question on implementing MSII-FM is how to choose the order of polynomials in the power-series approximation. This section suggests a data-driven block-deletion rule that can determine the order of polynomials. We first assume that  $\mathcal{K}(m)$  takes the form of  $\mathcal{K}(m) = \bar{\mathcal{K}}m^{\bar{\nu}}$  for some constants  $\bar{\mathcal{K}} \in (0, \infty)$  and  $\bar{\nu}$  satisfying the range of  $\nu$  in Assumption 6. Because

$$\min \left\{ \frac{2}{4d_3 + 3}, \frac{2}{4d_3^2 - d_3} \right\} = \begin{cases} 1/7 & \text{for } d_3 = 2 \\ 2/33 & \text{for } d_3 = 3 \end{cases},$$

we set the divergence rate of  $\mathcal{K}(m)$  equal to

$$\bar{\nu} = \begin{cases} 1/8 & \text{for } d_3 = 2 \\ 1/17 & \text{for } d_3 = 3 \end{cases}.$$

Also observe that the total number of terms in the  $q$ th-order polynomial constructed from  $d_3$  variables can be expressed as  $(1/d_3!) \prod_{k=1}^{d_3} (q+k)$ . After setting the proportionality constant equal to

$$\bar{\mathcal{K}} = \begin{cases} 4 & \text{for } d_3 = 2 \\ 8 & \text{for } d_3 = 3 \end{cases},$$

we can determine the order  $q^*$  via

$$\frac{1}{d_3!} \prod_{k=1}^{d_3} (q^* + k) \leq \bar{\mathcal{K}}m^{\bar{\nu}} < \frac{1}{d_3!} \prod_{k=1}^{d_3} (q^* + 1 + k).$$

Under such  $q^*$ ,  $K^* = (1/d_3!) \prod_{k=1}^{d_3} (q^* + k)$  indeed satisfies  $K^* \leq O(m^{\bar{\nu}})$ .